ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge...
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2017 /2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAREXAMINATION

FOR THE DEGREE OF BACHELOR OF APPLIED STATISTICS WITH COMPUTING

COURSE CODE: **STA 112**

COURSE TITLE: **INTRODUCTION TO PROBABILITY AND STATISTICS II**

DATE: 24TH APRIL, 2018 **TIME: 9AM – 12.00 NOON**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 4 PRINTED PAGES **PLEASE TURN OVER**

STA 111: INTRODUCTION TO PROBABILITY AND STATISTICS II

STREAM: ASC **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer Question **TWO** questions in section Aand any other **THREE** questions in section B.*

ii. *Maps and diagrams should be used whenever they serve to illustrate the answer.*

iii. *Do not write on the question paper.*

SECTION A (31 marks): Answer ALL questions.

QUESTION ONE (16 marks)

a) List and explain two main uses of poison distribution [2Marks]

b) A continuous random variable X has a pdf as follows

$$f(x) = C(x^2 - 2x + 3) \quad \text{for } 0 \leq x \leq 2$$

0 , elsewhere

i) Find C [2Marks]

ii) Find $\Pr(X \leq 1)$ [2Marks]

c) Let X be a random variable of continuous type, defined by the pdf f(x)

$$f(x) = \frac{2}{x^3} 1 < x < \infty$$

0, elsewhere

Find the C.D.F of the random variable X [3Marks]

d) Let X be a random variable with probability density function f(x).

$$f(x) = \frac{1}{18}(2x + 3) 2 \leq x \leq 4$$

0, else where

Obtain mean and variance of x [4Marks]

e) List three properties of expectation [3Marks]

QUESTION TWO (15 marks)

a) Consider the function $f(x) = \lambda e^{-\lambda x}$ for x>0

0, elsewhere

i) Obtain the m.g.f. of X hence [4Marks]

ii) Obtain the mean and the variance of X [5Marks]

b) For every 100,000 employees, with a poison distribution we expect an accident rate of 2.2 per 10,000 employees. Find $P(x \geq 1)$ [3Marks]

c) If for a random variables $x_1, x_2 x_3, \ldots, x_n$ occur with frequencies $f_1, f_2, f_3, \ldots, f_n$. Show that the second moment about the mean is given by $M_2 = M'_2 - (M'_1)^2$ [3Marks]

SECTION B (39 marks):

Answer any THREE questions. All Questions carry equal marks

QUESTION THREE (13 marks)

a) Given that a random variable X has the distribution with $f(x) = \begin{cases} \frac{1}{5-a}, & a < x < 5 \\ 0, & \text{elsewhere} \end{cases}$

i) Show that f(x) is a p.d.f. ? [3Marks]

ii) Find the mean and variance of X [6Marks]

b)) Given that a random variable X follows a Bernoulli distribution, obtain the first two raw moments [4Marks]

QUESTION FOUR (13 marks)

a) An unbiased coin is thrown three times. If the random variable X denotes the number of heads obtained, find the cumulative density function of X. [4Marks]

b) If the masses of 300 students are normally distributed with mean 68kg and standard deviation 3kg. How many students have massgreater than 72kg [4Marks]

c) Let us assume telephone calls to a toll-free 800 number are made in accordance with the poison process assumption at the rate of 120 per hour during the period 9 A.M to 12 noon. Find the probability of at least one call being made during any given;

i) Minute [2Marks]

ii) Second [3Marks]

QUESTION FIVE (13 marks)

The probability distribution function of Poisson distribution with parameter $\lambda$is given by

$$f(x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x=0,1,2\cdots\\ 0, & \text{elsewhere}\end{cases}$$

Find the moment generating function. Hence find its expectation and variance using the moment generating function [13 Marks]

QUESTION SIX (13 marks)

a) A college professor, based on his past experience, feels that there is a probability of 0.001 that he will be late to any given class and that being late or not for any class has no effect whether or not he is late for any other class. Then the number of times X that he will be late to his next 100 classes is a binomial random variable with parameters n=100 and p=0.001. Find the exact probabilities that he is late exactly 0 times and exactly 1 times.

[3Marks]

b) The probability distribution function of a beta distribution f(x) is given as;

$$f(x)=\begin{cases}\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} & 0<x<1\ \ \alpha,\beta>0\\ 0, else\ where\end{cases}$$ [10Marks]

QUESTION SEVEN (13 marks)

a) If C is a constant, show that the m.g.f of cx is $M_X(ct)$ [3Marks]

b) Let X be a normal distribution random variable. i.e. $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\ -\infty<x<\infty$ Show that it is a p.d.f. [7Marks]

c) The variable X can assume the value r with probability function $\frac{1}{4}(\frac{3}{4})^r$ for r=0, 1, 2… Show that X is a discrete random variable. [3Marks]

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