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OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2018 /2019 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS) / B.ED (ARTS)

COURSE CODE: MAT 216

COURSE TITLE: REAL ANALYSIS I

DATE: 10TH DECEMBER, 2018 **TIME: 9.00 AM – 12.00 PM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 4 PRINTED PAGES PLEASE TURN OVER

MAT 216

REGULAR-MAIN EXAM

MAT 216: REAL ANALYSIS I

STREAM:BED(Arts/Bsc/App.stat) DURATION:3 Hours

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INSTRUCTION TO CANDIDATES

i) Answer **ALL** questions in **SECTION A** and any other **THREE** questions in **SECTION B**.

ii) Do not write on the question paper.

SECTION A: [31 MARKS]

Question One : [16 marks]

a) Show that;

i) If A is a subset of an empty set $\emptyset$ then $A = \emptyset$ 3mks

ii) $(A \backslash B) \bigcap B = \emptyset$ 4mks

ii) $(A \bigcap B)^c = A^c \bigcup B^c$ 3mks

b) Let the function $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$. Find;

i) $(fog)(2)$ 3mks

ii) $g^{-1}(3)$ 3mks

Question Two : [15 marks]

a) i) Let (X, d) be a metric space. Then show that every finite subset A of X is closed. 2mks

ii) Construct a bounded set of real numbers with exactly 3 limit points. 3mks

b) Define the following concepts as applied to a set S of real numbers

i) Supremum 2mks

ii) Infimum 2mks

iii) Maximum 2mks

c) Show from first principles that the sequence $x_n = 1 + (-1)^n \frac{1}{n^2}$, $n \in \mathbb{N}$ converges to 1 in $\mathbb{R}$. 4mks

SECTION B: [39 MARKS]

Question Three : [13 marks]

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by;

$$f(x) = \begin{cases} 3 - x, & \text{if } x > 1; \\ 1, & \text{if x=1;} \\ 2x, & \text{if } x < 1. \end{cases}$$

i) Sketch the graph of f and find $f(1^+)$ and $f(1^-)$ 6mks

ii) Why is f not continuous at $x = 1$? 2mks

iii) State with reasons, whether $x = 1$ is a discontinuity of first kind or second kind. 5mks

Question Four : [13 marks]

a) Let (X, d) be a metric space and $E \subseteq X$. Show that :

i) E is closed in (X, d) if and only if $\overline{E} = E$. 5mks

ii) E is open in (X, d) if and only if $E^0 = E$. 5mks

b) Define a metric space 3mks

Question Five : [13 marks]

a) Use the mean value theorem to show that the equation

$x^3 - 4x = 0$

has at least one real root between -3 and -1. 7mks

b) Show that if t is irrational , then $S = \frac{t}{t+1}$ is irrational. 6mks

Question Six : [13 marks]

a) Define a bijective function and give an example by illustration. 4mks

b) Let (X, d) be a metric space and $(x_n)_{n=1}^{\infty}$ be a sequence of elements of X. Show that if $(x_n)_{n=1}^{\infty}$ converges in X then its limit is unique. 5mks

c) Show that $\sqrt{3}$ is irrational. 4mks

Question Seven : [13 marks]

a) For any $x, y \in R^k$ define the function $d(x, y)$ by

$d(x, y) = \sum_{i=1}^{k} |x_i - y_i|$. Show that d is a metric space on $\mathbb{R}^k$. 10mks

b) Given $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 5, 7, 9, 11\}$, find:

i) $A \bigcup B$ 2mks

ii) $A \bigcap B$ 1mks

END