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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2018 /2019 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: **STA 216**

COURSE TITLE: **MATHEMATICAL STATISTICS II**

DATE: 15/4/2019 **TIME: 9.00 AM-12.00 PM**

INSTRUCTION TO CANDIDATES

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STA 216: MATHEMATICS STATISTICS II

STREAM: ASC **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

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SECTION A (31 marks): Answer ALL questions.

QUESTION ONE (15 Marks)

a. If X and Y are independent random variables with joint probability density

$$f(x,y)=\begin{cases}\dfrac{k}{1+X^2+Y^2+X^2Y^2}, & -\infty<X<\infty, \quad -\infty<Y<\infty\\ 0, & elsewhere\end{cases}$$

i. Find k (4 marks)

ii. Determine if X and Y are independent (2marks)

b. Let Y_1, Y_2, Y_3 , Y_4 denote order statistics of order 4 from a distribution having pdf

$$f(x)=\begin{cases}2x, & 0<x<1\\ 0, & elsewhere\end{cases}$$

Find

i) $f_{Y_1}(y), f_{Y_3}(y)$ (6 marks)

ii) $f_{Y_1Y_2}(x,y)$ for $x<y$ (3 marks)

QUESTION TWO (16 Marks)

Let Y_1, Y_2, and Y_3 have a joint pdf $Y_1Y_2Y_3 \in \mathbb{R}^3$

$$f(Y_1,Y_2,Y_3)=\begin{cases}\dfrac{1}{(2\pi)^{3/2}}e^{-1/2(Y_1{}^2+Y_2{}^2+Y_3{}^2)}, & -\infty<Y_1<\infty, -\infty<Y_2<\infty, -\infty<Y_3<\infty\\ 0, & elsewhere\end{cases}$$

i. Find the marginal density functions $f(Y_1)$, $f(Y_2)$ and $f(Y_3)$ (6 marks)

ii. Find the conditional density function $f(Y_2/Y_1\,Y_3)$, $f(Y_3/Y_1\,Y_2)$. Comment on the density considering (i) above (10 marks)

SECTION B (39Marks)

QUESTION THREE (13Marks)

Let X_1, be distributed as $Gamma\ (\alpha, 1)$ and X_2 be distributed as $Gamma\ (\beta, 1)$ and further X_1 and X_2 are independent. Let $Y_1 = \frac{X_1}{X_1+X_2}$ and $Y_2 = X_1 + X_2$.

i. Find the joint distribution of Y_1 and Y_2 (6marks)
ii. Find the distribution of Y_2 (6 marks)
iii. Are Y_1 and Y_2 independent? (1 mark)

QUESTION FOUR (13Marks)

a. If X follows a $binomial(n, \pi)$ distribution obtain its characteristic function. (3 marks).

b. i. Given that X is a discrete random variable with mean μ *and* variance ${\sigma_x}^2$ show that

$$Pr(|X - \mu|) \geq k\sigma_x) \leq \frac{1}{k^2}$$ (5 marks)

ii. Hence find the lower bound for $Pr(17 < X < 33)$ and the upper bound for the $Pr(x > 33)$, given that the mean X =25 and variance =16. (5 marks)

QUESTION FIVE (13Marks)

a. Obtain the moment generating function for the exponential distribution with parameter λ, i.e.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & elsewhere \end{cases}$$

(5 marks)

b. Suppose X and Y are independent random variables with Exp(λ)distributions.

Using the transformation rule, find the joint density function of U = X + Y. and V= X (8 marks)

QUESTION SIX (13Marks)

Let Y_1, Y_2, and Y_3 be independent standard normal random variable:

$X_1 = Y_1$

$X_2 = \frac{Y_1 + Y_2}{2}$

$X_3 = \frac{Y_1 + Y_2 + Y_3}{3}$

i. Obtain the joint pdf of X_1, X_2 and X_3 (6 marks)

ii. Obtain the marginal density function of X_2 (7 marks)

QUESTION SEVEN (13MARKS)

a) Show the proof of existence of a characteristic function (5marks)

b. Given that $X_1, X_1, \ldots, X_n$ are identically and independent random variables from $Bernoulli(p)$. Define Y= $X_1 + X_1 +, \ldots, +X_n$. Find the distribution of Y, using the moment generating function technique. (8 marks)