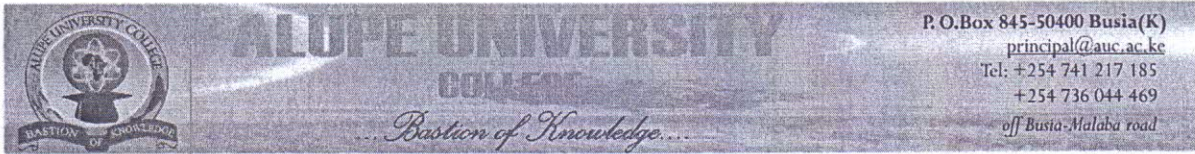


STA 216



OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2018 /2019 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH
COMPUTING)

COURSE CODE: STA 216

COURSE TITLE: MATHEMATICAL STATISTICS II

DATE: 15/4/2019

TIME: 9.00 AM-12.00 PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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STA 216: MATHEMATICS STATISTICS II

STREAM: ASC

DURATION: 3 Hours

INSTRUCTION TO CANDIDATES

Answer ALL questions from section A and any THREE from section B.

SECTION A (31 marks): Answer ALL questions.

QUESTION ONE (15 Marks)

a. If X and Y are independent random variables with joint probability density

$$f(x, y) = \begin{cases} \frac{k}{1 + X^2 + Y^2 + X^2Y^2}, & -\infty < X < \infty, \quad -\infty < Y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- i. Find k (4 marks)
- ii. Determine if X and Y are independent (2marks)

b. Let Y_1, Y_2, Y_3, Y_4 denote order statistics of order 4 from a distribution having pdf

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

- i) $f_{Y_1}(y), f_{Y_3}(y)$ (6 marks)
- ii) $f_{Y_1Y_2}(x, y)$ for $x < y$ (3 marks)

QUESTION TWO (16 Marks)

Let $Y_1, Y_2,$ and Y_3 have a joint pdf $Y_1Y_2Y_3 \in \mathbb{R}^3$

$$f(Y_1, Y_2, Y_3) = \begin{cases} \frac{1}{(2\pi)^{3/2}} e^{-1/2(Y_1^2 + Y_2^2 + Y_3^2)}, & -\infty < Y_1 < \infty, -\infty < Y_2 < \infty, -\infty < Y_3 < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- i. Find the marginal density functions $f(Y_1), f(Y_2)$ and $f(Y_3)$ (6 marks)
- ii. Find the conditional density function $f(Y_2/Y_1, Y_3)$, $f(Y_3/Y_1, Y_2)$. Comment on the density considering (i) above (10 marks)

SECTION B (39Marks)**QUESTION THREE (13Marks)**

Let X_1 be distributed as *Gamma* ($\alpha, 1$) and X_2 be distributed as *Gamma* ($\beta, 1$) and further X_1 and X_2 are independent. Let $Y_1 = \frac{X_1}{X_1 + X_2}$ and $Y_2 = X_1 + X_2$.

- i. Find the joint distribution of Y_1 and Y_2 (6marks)
- ii. Find the distribution of Y_2 (6 marks)
- iii. Are Y_1 and Y_2 independent? (1 mark)

QUESTION FOUR (13Marks)

- a. If X follows a *binomial*(n, π) distribution obtain its characteristic function. (3 marks).
- b. i. Given that X is a discrete random variable with mean μ and variance σ_x^2 show that

$$Pr(|X - \mu| \geq k\sigma_x) \leq \frac{1}{k^2} \quad (5 \text{ marks})$$

- ii. Hence find the lower bound for $Pr(17 < X < 33)$ and the upper bound for the $Pr(x > 33)$, given that the mean $X = 25$ and variance = 16. (5 marks)

QUESTION FIVE (13Marks)

- a. Obtain the moment generating function for the exponential distribution with parameter λ , i.e.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

(5 marks)

- b. Suppose X and Y are independent random variables with $\text{Exp}(\lambda)$ distributions.

Using the transformation rule, find the joint density function of $U = X + Y$ and $V = X$ (8 marks)



QUESTION SIX (13Marks)

Let $Y_1, Y_2,$ and Y_3 be independent standard normal random variable:

$$X_1 = Y_1$$

$$X_2 = \frac{Y_1 + Y_2}{2}$$

$$X_3 = \frac{Y_1 + Y_2 + Y_3}{3}$$

- i. Obtain the joint pdf of X_1, X_2 and X_3 (6 marks)
- ii. Obtain the marginal density function of X_2 (7 marks)

QUESTION SEVEN (13MARKS)

- a) Show the proof of existence of a characteristic function (5marks)
- b. Given that X_1, X_1, \dots, X_n are identically and independent random variables from *Bernoulli*(p). Define $Y = X_1 + X_1 + \dots + X_n$. Find the distribution of Y , using the moment generating function technique. (8 marks)