



**ALUPE UNIVERSITY**

**COLLEGE**

*Bastion of Knowledge...*

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OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, STUDENT AFFAIRS AND RESEARCH

# UNIVERSITY EXAMINATIONS

## 2018 /2019 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION

**FOR THE DEGREE OF BACHELOR OF SCIENCE  
IN APPLIED STATISTICS WITH COMPUTING**



**COURSE CODE: STA 212**

**COURSE TITLE: MATHEMATICAL STATISTICS 1**

**DATE: 13 DECEMBER, 2018**

**TIME: 9.00 AM – 12.00 PM**

### INSTRUCTION TO CANDIDATES

- SEE INSIDE

**THIS PAPER CONSISTS OF 4 PRINTED PAGES**

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## STA 212: MATHEMATICAL STATISTICS 1

STREAM: BSc (Applied Statistics with Computing)

DURATION: 3 Hours

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**INSTRUCTION TO CANDIDATES**Answer **ALL** questions from section A and **ANYTHREE** Questions in section B.

All questions in section B carry Equal Marks

Duration of the examination: 3 hours

## =====

**QUESTION ONE (16 Marks)**

a) Let X and Y have joint pdf

$$f(x, y) = \begin{cases} k(x + y)I_{(0,1)}(x)I_{(0,1)}(y) \\ 0, & \text{elsewhere} \end{cases}$$

- i. Obtain  $k$  [3Mks]
  - ii. Determine if X and Y are independent [3Mks]
  - iii. Find  $E(Y/x)$  [3Mks]
- b) If X and Y have joint pdf

$$f(x, y) = \begin{cases} 4 \exp\{-2(x + y)\}, & 0 < x < \infty, 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- i) Obtain the joint moment generating function  $M_{X,Y}(t_1, t_2)$  [3Mks]
- ii) Hence or otherwise show that  $cov(X, Y) = 0$  [3Mks]
- iii) Find the distribution of X [1Mk]

**QUESTION TWO (15Marks)**

In an experiment three fair coins are tossed. If X is the number of heads in the first and third coins and Y is the number of heads from the second and third coins. Find the;

- a) Joint probability mass function of X and Y [5Mks]
- b) Correlation between X and Y [5Mks]
- c) Joint cumulative distribution function  $F_{X,Y}(x, y)$  [5Mks]

**SECTION B (39 marks)**

**QUESTION THREE (13marks)**

- a) Let X and Y be two continuous random variables with joint pdf  $f(x, y)$ . Prove that X and Y are stochastically independent if and only if  $f(x, y)$  can be written as a product of non-negative function of x and non-negative function of y. [5Mks]
- b) Let X and Y be two random variables such that

$$f_X(x) = I_{(0,1)}(x)$$

$$f(y/x) = \binom{n}{y} x^y (1-x)^{n-y}, y=0,1,2,\dots,n$$

- i) Find  $f_Y(y)$  [5Mks]
- ii) Workout  $E[E Y/x]$  [3Mks]

**QUESTION FOUR(13marks)**

- a) If (X,Y) follows a bivariate normal distribution(assume usual notation), show that the conditional density of Y given X is also normal. [7Mks]
- b) Let X denote the height in centimeter and Y the weight in kilograms of male college students. Assume that X and Y have a bivariate normal distribution with parameters  $\mu_x = 185, \mu_y = 84, \sigma_x^2 = 100, \sigma_y^2 = 64$  and  $\rho = \frac{3}{5}$
- i) Obtain the conditional distribution of Y, given  $X=190$  [2Mks]
- ii) Find  $\Pr(86.4 < Y < 95.36/X = 190)$  [4Mks]

**QUESTION FIVE(13marks)**

If the joint density function of X and Y is given by

$$f(x, y) = \begin{cases} cxy, & 2 < x < y < 4 \\ 0, & \text{elsewhere} \end{cases}$$

- i) Find c [3Mks]
- ii) Obtain  $f(y/x)$  [4Mks]
- iii) Work out  $Var(Y/x)$  [6Mks]



**QUESTION SIX(13marks)**

If X and Y have the trinomial distribution

$$f(x, y) = \frac{n!}{x! y! (n-x-y)!} p^x q^y (1-p-q)^{n-x-y}$$

For  $x, y = 0,1,2, \dots, n$  and  $x + y \leq n, 0 \leq p, 0 \leq q, p + q \leq 1$

- i) Find the conditional density of Y given X [5Mks]

- ii) Obtain the correlation between X and Y

[8Mks]

**QUESTION SEVEN(13marks)**

If X and Y have the joint density

$$f(x, y) = \begin{cases} c \exp - \frac{1}{2} (2x^2 + y^2 - 4x - 4y + 6) & -\infty < x < \infty, -\infty < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

- i) Find c [4Mks]  
ii) Find  $f(y/x)$  [4Mks]  
iii) Find  $f(y)$  [3Mks]  
iv) From your answers in parts ii) and iii) above what can you say about the correlation between X and Y [2Mks]

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