

# OFFICE OF THE DEPUTY PRINCIPAL ACADEMICS, STUDENT AFFAIRS AND RESEARCH

#### **UNIVERSITY EXAMINATIONS**

#### **2018 /2019 ACADEMIC YEAR**

# FIRST YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

**COURSE CODE:** 

**STA 115** 

**COURSE TITLE:** 

INTRODUCTION TO MATHEMATICS FOR FINANCE

DATE: 24/4/2019

TIME: 2.00 PM - 5.00 PM

## **INSTRUCTION TO CANDIDATES**

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#### STA 115: INTRODUCTION TO MATHEMATICS FOR FINANCE

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STREAM: ASC DURATION: 3 Hours

#### INSTRUCTION TO CANDIDATES

Answer ALL questions from section A and any THREE from section B.

SECTION A (31 marks): Answer ALL questions.

# **QUESTION ONE (16 Marks)**

- a) Discuss the following terms as used in finance
  - i) Interest [1Mark]
  - ii) Breakeven point [1Mark]
  - iii) Mortgage [1Mark]
  - iv) Market Equilibrium [1Mark]
- b) i) Differentiate between domain and range of the function [2Marks]
  - ii) Given  $f(x) = 2x^2 + 5x + 1$ , find the value of f when x = 3; x = 2; [2Marks]
- c) If the amount of £2000 was invested 3 years ago to have amount of £4000 now. At what rate is the amount discounted? [3Marks]
- d) i) Show how the calculation of compound interest is an application of exponential function [2Marks]
  - ii) Shadrack invests a bonus of £9000 at 6% annual interest compounded semiannually for 4 years. How much interest will he earn? [3Marks]

# **QUESTION TWO (15Marks)**

- a) i) Give the difference between Nominal interest and effective interest [1Mark]
  - ii) What nominal rate, compounded quarterly, is equivalent to an effective annual rate of 10%? [2Marks]
- b) Biologists studying salmon have found that the oxygen consumption of yearling salmon (in appropriate units) increases exponentially with the speed of swimming according to the function defined by  $f(x) = 100e^{0.6x}$  where x is the speed in feet per second. Find the following.

i) The oxygen consumption when the fish are still

[2Marks]

ii) The oxygen consumption at a speed of 2 ft per second

[2Marks]

c) Let matrix A be

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$
 Find the inverse of **A**

[4Marks]

d) Solve the following equation;  $\log x + \log (x - 3) = 1$ , (assume base=10)

[2Marks]

e) Example 6 (Electricity Cost Function) Electricity is charged to consumers at the rate of 10 cents per unit for the first 50 units and 3 cents per unit for amount in excess of this. Find the function c(x) that gives the cost of using x units of electricity. [2Marks]

## SECTION B (39 marks)

# **QUESTION THREE (13MKS)**

a) Define the term perpetuity in finance?

[2Marks]

- b) Investment A: done by making payments of amount £50,000/= at the end of every year for 15 years at the rate of interest of 8%.
  - Investment B: made by payments of £60,000 at the beginning of every year for 13 years at an interest rate of 8%.
  - If both investments are thereafter accumulated up to the end of the 20<sup>th</sup> year after which they are converted to perpetuity to be received at the end of every year. Determine which perpetuity gives a higher amount? [4Marks]
- c) If you deposit £5000 into an account paying 6% annual interest compounded monthly, how long until there is £8000 in the account? [3Marks]
- d) Given the series payments of amount £2,000 made at the end of every year for ten years at an annual effective rate of 10%. Determine
  - i) Present value of the series

[2Marks]

ii) Accumulated value of the series of payments at t=10

[2Marks]

#### **QUESTION FOUR (13MKS)**

A magazine article argued that the cause of the obesity epidemic is the decreasing cost of food (in real terms) due to the increasing surplus of food. As one piece of evidence, the following table was provided, which we have updated, showing. corn production (in billions of bushels) for selected years.

Year	Production (Billions of Bushels)	
1930	1.757	
1940	2.207	
1950	2.764	
1960	3.907	
1970	4.152	



1980	6.639	
1990	7.934	
2000	9.968	
2005	11.112	

a) Plot the data. Does the production appear to grow linearly or exponentially? [6Marks]

b) Find an exponential function in the form of  $p(x) = p_0 a^{x-1930}$  that models this data, where x is the year and p(x) is the production of corn. Use the data for 1930 and 2005. Plot the function on the graph in (a) [5Marks]

c) Determine the expected annual percentage increase in corn production during this time period.

[2Marks]

# **QUESTION FIVE (13MKS)**

- a) Define the following
  - i) Feasible region
  - ii) Constraints
  - iii) Feasible solution

[1Mark]

[1Mark]

[1Mark]

- b) A flight leaves New York at 8 P.M. and arrives in Paris at 9 A.M. (Paris time). This 13-hour difference includes the flight time plus the change in time zones. The return flight leaves Paris at 1 P.M. and arrives in New York at 3 P.M. (New York time). This 2-hour difference includes the flight time minus time zones, plus an extra hour since flying westward is against the wind. Find the actual flight time eastward and the difference in time zones. [4Marks]
- c) A restaurant owner orders a replacement set of knives, forks, and spoons. The box arrives containing 40 utensils and weighing 141.3 oz (ignoring the weight of the box). A knife, fork, and spoon weigh 3.9 oz, 3.6 oz, and 3.0 oz, respectively.
  - (i) How many solutions are there for the number of knives, forks, and spoons in the box? [3Marks]
  - (ii) Find the solution with the smallest number of spoons.

[3Marks]

#### **QUESTION SIX (13MKS)**

- a) Give the difference between the following:
  - i) Annuity due and annuity immediate

[2Marks]

ii) Row echelon form of a matrix and rank of a matrix

[2Marks]

b) Use the Gauss-Jordan method to solve the system

[9 Marks]

$$x + 5z = -6 + y$$
  
 $3x + 3y = 10 + z$   
 $x + 3y = 10 + z$ 

## **QUESTION SEVEN (13MKS)**

a) Find an equation of the line through (5,4) and (-10, -2).

[3Marks]

b) A Farmer raises only goats and pigs. She wants to raise no more than 16 animals, including no more than 10 goats. She spends \$25 to raise a goat and \$75 to raise a pig, and she has \$900 available for this project. Each goat produces \$12 in profit and each pig \$40 in profit. How many goats and how many pigs should she raise to maximize total profit? Sketch the feasible region for the following set of constraints, and then find the maximum and minimum values of the objective function. [10Marks]

