ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR ACADEMICS,

RESEARCH AND STUDENT AFFAIRS

UNIVERSITY

EXAMINATIONS 2023/2024

ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: MAT 104

COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

DATE: 6TH DECEMBER 2023 **TIME:** 9.00AM - 12.00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 4 PRINTED PAGES

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INSTRUCTION TO CANDIDATES

i. *Answer ALL Questions from Section A and any THREE from Section B*
ii. *Do not Write on the Question Paper*
iii. *Answers Should be Comprehensive, Informative and Neat*

SECTION A (31 MARKS): Answer ALL Questions in this Section

QUESTION ONE (16 MARKS)

a) Define the terms:
- **(i)** Median as referred to in geometry **(1 mark)**
- **(ii)** Momentum **(1 mark)**

b) Find the equation of the line through the point $(1,2)$ perpendicular to the line $3x+4y=7$ **(2 marks)**

c) Express the equation $x^2=4y$ in polar coordinates **(2 marks)**

d) ΔOAB has vertices $(0,0),(-3,9)$ and $(12,4)$. Show that ΔOAB is a right angle **(3 marks)**

e) Determine if the sets of vectors below are parallel or not: $\vec{a}=(2,-4,1)$ and $\vec{b}=(-6,12,-3)$ **(2 marks)**

f) Determine the centre and the radius of the circle passing through points $(4,3),(0,1)$ and $(1,0)$ **(5 marks)**

QUESTION TWO (15 MARKS)

a) Write the line L through the point $P(3,4,5)$ and parallel to the vector $V=(5,-2,7)$ in:
- **(i)** Vector form **(1 mark)**
- **(ii)** Parametric form **(1 mark)**
- **(iii)** Symmetric form **(1 mark)**

b) Find the equation of the plane containing the points $A(6,-7,-3)$, $B(3,-3,2)$ and $C(7,4,2)$ **(5 marks)**

c) Find the length of the curve $y=10\cosh\frac{x}{10}$, between $x=-1$ and $x=2$ **(7 marks)**

SECTION B (39 MARKS): Answer any THREE Questions from this Section

QUESTION THREE (13 MARKS)

a) A $325kg$ motorcycle is moving at $140km/h$.
- **(i)** Find its momentum **(2 marks)**

(ii) At what velocity is the momentum of $1754kg$ car equal to that of the motorcycle? **(3 marks)**

b) Find the area enclosed between the curves $r_1 = 1+\sin\theta$ and $r_2 = 3\sin\theta$ **(4 marks)**

c) A model of a car moves from a circular track of radius $0.4m$ at 2 revolutions per second. What is its:

(i) Period T **(2 marks)**

(ii) Angular velocity ω **(2 marks)**

QUESTION FOUR (13 MARKS)

a) A stone is projected vertically upwards with a velocity of $30ms^{-1}$ from the ground. Calculate:

(i) the time it takes to reach maximum height **(2 marks)**

(ii) maximum height **(2 marks)**

(iii) velocity with which it lands on the ground **(2 marks)**

b) Determine the angle between $\vec{A} = (3,-4,-1)$ and $\vec{B} = (0,5,2)$ **(4 marks)**

c) Find the projection of the vector $\vec{A} = i - 2j + k$ on the vector $\vec{B} = 4i - 4j + 7k$ **(3 marks)**

QUESTION FIVE (13 MARKS)

a) Obtain the equation of the plane with normal vector $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ containing the point $(4,0,-3)$ **(4 marks)**

b) Find the distance between the parallel planes whose equations are $4x - 4y + 2z + 6 = 0$ and $6x - 6y + 3z + 4 = 0$ **(4 marks)**

c) Find the equation of the tangent to the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$ **(5 marks)**

QUESTION SIX (13 MARKS)

a) A ball of mass $35g$ travelling at $20ms^{-1}$ strikes a wall at right angles and rebounds with a speed of $16ms^{-1}$. Find the impulse exerted on the ball **(4 marks)**

b) Find the point of intersection between the following lines $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z-1}{-1}$ and $x + 3 = \frac{y-8}{-3} = \frac{z+3}{2}$ **(4 marks)**

c) Sketch the curve given parametrically by $x = \sin\theta, y = \sin 2\theta$. **(5 marks)**