ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS AFFAIRS

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE COMPUTER SCIENCE

COURSE CODE: MAT 111

COURSE TITLE: GEOMETRY AND ELEMENTARY APPLIED MATHEMATICS

DATE: 23RD APRIL, 2024 **TIME: 2 – 5PM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

REGULAR–MAIN EXAM

MAT 111: GEOMETRY AND ELEMENTARY APPLIED MATHEMATICS

STREAM: BSc ASC/CS DURATION: 3 Hours

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***

ii. *Do not write on the question paper.*

iii. Take $g = 9.8m/s^2$

No sharing of scientific calculators.

Do not write on this question paper.

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SECTION A: ANSWER ALL QUESTIONS [31 MARKS]

QUESTION ONE (16 MARKS)

a) Two lines are described as follows; the first has gradient -1 and passes through the point $R(2,1)$; the second passes through the two points with coordinates $P(2,0)$ and $Q(0,4)$. Find the equation of both lines and find the coordinates of their point of intersection. **(4mks)**

b) Express the polar equation $r = 2 + 2\cos\theta$ in rectangular form **(2mks)**

c) Two planes **P** and **Q** have equations $5x + y - z = 4$ and $x - 3y + 6z = 4$ respectively, determine;
   i. The acute angle between the planes, correct to the nearest degree. **(2mks)**
   ii. The symmetric equations for the line of intersection L of these two planes **(3mks)**

d) State and prove the Newton's Second laws of motion. **(2mks)**

e) A body of mass 2kg moving with speed 5m/s collides directly with another of mass 3 kg moving with speed 4m/s in the same direction. The coefficient of restitution is $^2/_3$. Find the velocities after collision. **(3mks)**

QUESTION TWO (15 MARKS)

a) Given the vectors $\boldsymbol{u} = \boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{v} = -4\boldsymbol{i} + 3\boldsymbol{j} - 11\boldsymbol{k}$, $\boldsymbol{w} = 3\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$.
   i. Find the volume generated by the parallelepiped determined by the three vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$. **(2mks)**
   ii. What is the angle between $(\boldsymbol{u} \times \boldsymbol{v})$ and $\boldsymbol{w}$? **(2mks)**

b) John kicks a ball and the ball undergoes a projectile motion with an angle of 53^0 to horizontal. It's initial velocity is 10 m/s, find;
   i. The maximum height it can reach **(2mks)**
   ii. Horizontal displacement **(3mks)**
   iii. Time of flight **(2mks)**

c) Find the area enclosed by the curves $r_1 = 1 - \sin\theta$ & $r_2 = 3\sin\theta$ **(4mks)**

SECTION B: ANSWER ANY THREE QUESTIONS [39MARKS]

ESTION THREE (13 MARKS)

a) A ball is fired at an angle of 30^0 to the horizontal at the velocity of 25 m/s. Determine its time of flight and range. (Take g = 10 m/s^2). **(5mks)**

b) Sketch and identify the curve $r = Cos\,2\theta, 0 \leq \theta \leq 360$. **(4mks)**

c) Find the work done by the force $F = 3i + 4j - k$ on moving an object from a point (1, 0, 0) to another point (4, 3, 9). **(4mks)**

QUESTION FOUR (13 MARKS)

a) i. Find the equation of a plane passing through points $A(1, 2, 3)$, $B(3, 2, 1)$ and $C(-1, -2, 2)$ **(5mks)**

ii. Find the distance from the point (3, 2, 1) to the plane $10x + 2y - 2z = 5$. **(3mks)**

b) A 2kg block A is released from rest falls a distance $h = 0.5m$ and strikes plate B (3kg mass). The coefficient of restitution between A and B is $e = 0.6$, and the spring's stiffness is $k = 30N/m$. Find the velocity of block A just after the collision. **(5mks)**

QUESTION FIVE (13 MARKS)

a) Calculate the length of the spiral $r = e^{\theta}$ between $0 \leq \theta \leq 1$. **(4marks)**

b) The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s in meters; **(9mks)**

i. Find the velocity at time t
ii. What is the velocity after $2s$
iii. When is the particle at rest
iv. When is the particle moving forward
v. Find the total distance travelled by the particle during the first five seconds
vi. Find the acceleration at time t and after $4s$.

QUESTION SIX (13 MARKS)

a) Express the equation $x^2 = 4y$ in polar form. **(3mks)**

b) Find the point of intersection for $r_1 = 1 - \cos\theta$ and $r_2 = \cos\theta$ **(4mks)**

c) A 50 kg skier is pulled up a frictionless ski slope that makes an angle of 8° with the horizontal, by holding onto a ski rope that moves parallel to the slope. Determine the magnitude of the force of the rope on the skier at an instant when: **(6mks)**

i. the rope is moving with constant speed of $2.0\,ms^{-1}$
ii. when the rope is moving with a speed of $2.0\ ms^{-1}$, but that speed is increasing at a rate of $0.10\ ms^{-2}$.

QUESTION SEVEN (13 MARKS)

a) Define the term centre of gravity. **(1mks)**

b) A typical adult human has a mass of about 70.0 kg. Calculate the force a full moon can exert on such a human when it is directly overhead with its centre 380,000 km away. Compare this force with that exerted on the human by the earth. **(6mks)**

c) A uniform rod AB of mass 10 kg and length 6 m rests in equilibrium with A on rough horizontal ground. The rod is resting on a smooth peg at C, where AC = 4 m. The angle between AB and the ground is θ, where tan θ = 0.4. Given the rod is on the point of slipping. Find the coefficient of friction between the rod and the ground. **(6mks)**