MAT 216

ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR

ACADEMICS, RESEARCH AND STUDENTS' AFFAIRS

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS/SCIENCE

COURSE CODE: MAT 216

COURSE TITLE: REAL ANALYSIS I

DATE: 20TH DECEMBER 2023 **TIME:** 9.00AM-12.00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

STREAM: BED (Arts/Science) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from* ***section A*** *and ANY from* ***section B.***

ii. *Do not write on the question paper.*

SECTION A (31 Marks)

Answer ALL questions in this section.

QUESTION ONE (16 Marks)

a) Define the following terms
   i) Interior point (1 Mark)
   ii) Exterior point (1 Mark)
   iii) Boundary point (1 Mark)

b) Show that for every x, with $|x| < 1$, $\lim_{n\to\infty} nx^n = 0$. (5 Marks)

c) Apply the integral test to the series $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$. (5 Marks)

d) Discuss the convergence of the harmonic series. (3 Marks)

QUESTION TWO (15 Marks)

a) Show that the $\lim_{n\to 2} x^2 = 4$. (5 Marks)

b) Prove that a monotone sequence converges if and only if it is bounded. (5 Marks)

c) Let $x, y \in \mathbb{R}$, prove that $\|x \cdot y\| = \|x\|\|y\|$. (5 Marks)

SECTION B (39 Marks)

Answer ANY THREE questions.

QUESTION THREE (13 Marks)

a) Let $f: X \to Y$ and $g: Y \to Z$ such that $ran(f) \subseteq dom(g)$, show that
   i) If f and g are onto then so is the composition function $g \circ f$. (3 Marks)
   ii) If f and g are one-to-one then so is the composition function $g \circ f$. (4 Marks)

b) Prove that a sequence $\{x_n\}$ converges to zero if and only if the sequence $\{|x_n|\}$ converges to zero. (6 Marks)

QUESTION FOUR (13 Marks)

a) Show that the sequence $\{S_n\} = \frac{n+1}{n}$ is a Cauchy sequence. (7 Marks)

b) Let $\{s_n\}$ be the sequence which converges to s. Prove that any subsequences of $\{s_n\}$ converges to s. (6 Marks)

QUESTION FIVE (13 Marks)

a) Let $\{s_n\}$ and $\{t_n\}$ be sequences of real numbers which converges to s and t respectively. Prove that $\lim_{n\to\infty} s_n \cdot t_n = s \cdot t$. (8 Marks)

b) Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$. (5 Marks)

QUESTION SIX (13 Marks)

a) Show that $f(x) = \frac{1}{x}$ is continuous at $x = 1$. (6 Marks)

b) Prove that if two sets A and B are open then $A \cap B$ is open. (5 Marks)

c) State the least upper bound property of a set $S \subseteq \mathbb{R}$. (2 Marks)

QUESTION SEVEN (13 Marks)

a) Prove that given two sets A and B, if $A = B$ then $(A \subseteq B)\wedge(B \subseteq A)$. (6 Marks)

b) Prove that if a series $\sum|a_n|$ converges then the series $\sum a_n$ converges. (4 Marks)

c) Prove that a non-empty subset S of an ordered field φ can have at most one least upper bound. (3Marks)