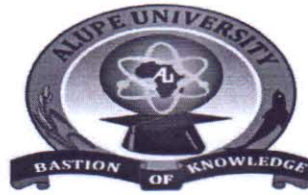# ALUPE UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR ACADEMICS,

RESEARCH AND STUDENT AFFAIRS

UNIVERSITY EXAMINATIONS 2023/2024 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

COURSE CODE: MAT 418

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 21ST DECEMBER 2023   **TIME: 9.00AM – 12.00PM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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INSTRUCTION TO CANDIDATES

i. *Answer ALL Questions from Section A and any THREE from Section B*

ii. *Do not Write on the Question Paper*

iii. *Answers Should be Comprehensive, Informative and Neat*

SECTION A (31 MARKS): Answer ALL Questions in this Section

QUESTION ONE (16 MARKS)

1. Define the terms:
   - a) Degree of a differential equation   (1 mark)
   - b) Order of a differential equation   (1 mark)
   - c) Semi-linear equation   (1 mark)

2. Find the PDE by eliminating the arbitrary constants a and b from the equation:

   $$z = ax + (1-a)y + b \qquad \text{(6 marks)}$$

3. Solve: $p + 3q = 5z + \tan(y - 3x)$ (7 marks)

QUESTION TWO (15 MARKS)

1. Find the general solution of the following equation: $z(z^2 + xy)(px - qy) = x^4$ (7 marks)

2. Form a PDE by eliminating the arbitrary functions f and g from:

   $$z = f(x + iy) + g(x - iy) \qquad \text{(8 marks)}$$

SECTION B (39 MARKS): Answer any THREE Questions from this Section

QUESTION THREE (13 MARKS)

a) Solve: $yzp + 2xq = xy$ (6 marks)

b) Prove that the complete integral of $z = px + qy + \dfrac{pq}{pq - p - q}$ represents all planes such that the

algebraic sum of the intercepts on these axes is unity   (7 marks)

QUESTION FOUR (13 MARKS)

a) Verify whether the following differential equation is integrable:

   $$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0 \qquad \text{(6 marks)}$$

b) Find the equation of the surface satisfying $4yzp + q + 2y = 0$ and passing through

   $$y^2 + z^2 = 1, x + z = 2 \qquad \text{(7 marks)}$$

QUESTION FIVE (13 MARKS)

a) Find the complete integral of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method **(7 marks)**

b) Find the PDE of all spheres of radius λ, having the centre in the $x - y$ plane **(6 marks)**

QUESTION SIX (13 MARKS)

a) Use Jacobi's method to find the complete integral of $p_1 x_1 + p_2 x_2 = p_3{}^2$ **(7 marks)**

b) Solve: $yz \log z\, dx - xz \log z\, dy + xy\, dz = 0$ **(6 marks)**

QUESTION SEVEN (13 MARKS)

a) Find a PDE by eliminating arbitrary function: $z = f(x^2 - y^2)$ **(6 marks)**

b) Solve the equation: $z(x + y)p + z(x - y)q = x^2 + y^2$ **(7 marks)**

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