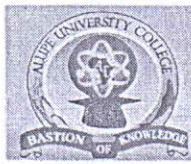


MAT 204



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... Bastion of Knowledge ...

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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF
EDUCATION ARTS AND SCIENCE

COURSE CODE: MAT 204

COURSE TITLE: LINEAR ALGEBRA II

DATE: 2ND FEB 2022 TIME: 2.00PM -5.00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES

PLEASE TURN OVER

INSTRUCTIONS TO CANDIDATES

- i. Answer ALL Questions from section A and any **THREE** from section B.
- ii. Do not write on the question paper.

SECTION A (31 Marks)

Answer ALL questions in this section.

Question One (16 Marks)

- a) Verify that the following is an inner product in P_2

$$\langle p(x), q(x) \rangle = a_0b_0 + a_1b_1 + a_2b_2$$

where $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$. (4 marks)

- b) Transform the set $S = \{1, x, x^2\}$ which is a basis for P_2 into an orthonormal basis using the Gram-Schmidt orthonormalization process with respect to the integral inner product on P_2 defined as

$$\langle p(x), q(x) \rangle = \int_{-1}^1 p(x)q(x)dx \quad (5 \text{ marks})$$

- c) Given $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation defined as $T(x, y) = (2x - 3y, x + y)$ and $B = \{(1, 2), (2, 3)\}$, $B' = \{(1, 3), (1, 4)\}$ are both bases for \mathbb{R}^2 . Find
- A i.e matrix of representation of T with respect to basis B.
 - Transition matrix P from B' to B. Hence use P and A to obtain A' (i.e. matrix of representation of T with respect to B'). (7 marks)

Question Two (15 Marks)

- a) Let $A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$ use the eigen value method to derive an explicit formula for A^n and solve the system of differential equations $\frac{dx}{dt} = 2x - 3y$ and $\frac{dy}{dt} = 4x - 5y$ given that $x = 7$ and $y = 13$ when $t = 0$. (6 marks)

- b) Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form, $x^2 + 2xy + y^2 - x + y = 0$ (6 marks)

- c) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 4 & -3 \\ 0 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix}$ by making use of the Cayley-Hamilton theorem.

(3 marks)

SECTION B (39 Marks)**Answer any THREE questions.****Question Three (13 Marks)**

- a) Consider the vector space of polynomials with inner product defined by

$$\langle f(x), g(x) \rangle = \int_{-1}^1 f(x)g(x)dx \text{ and the polynomials } f(x) = x \text{ and } g(x) = 1 + x. \text{ Find}$$

- i) $\langle f(x), g(x) \rangle$
 - ii) $\|f\|$
 - iii) $\|g\|$
 - iv) Normalize f and g . (6 marks)
- b) State and prove the Cauchy - Schwarz inequality. (7 marks)

Question Four (13 Marks)

a) For the matrix $A = \begin{pmatrix} 2 & 0 & -2 \\ -1 & 2 & -1 \\ -2 & 0 & 2 \end{pmatrix}$

- i) write down the characteristic polynomial
- ii) write down the characteristic equation
- iii) find the eigen values and eigen vectors corresponding to each eigen value.
- iv) find the basis for each eigen space. (10 marks)

- b) Find the minimal polynomial $m(\lambda)$ of the matrix

$$A = \begin{pmatrix} \alpha & \beta \\ 0 & \alpha \end{pmatrix} \quad (3 \text{ marks})$$

Question Five (13 Marks)

- a) If λ_1 and λ_2 are eigen values of a symmetric matrix A associated with eigen vectors v_1 and v_2 respectively, show that v_1 and v_2 are orthogonal. (4 marks)
- b) If v_1, v_2, \dots, v_n are eigenvectors associated with distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ of a matrix $A_{n \times n}$ then show that the set $\{v_1, v_2, \dots, v_n\}$ is linearly independent. (6 marks)

- c) Determine if $B = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ is orthogonal matrix (3 marks)

Question Six (13 Marks)

- a) Let V be the vector space of polynomials over \mathbb{R} and define $\langle f(t), g(t) \rangle = \int_0^1 f(t)g(t)dt$.
 Find the angle θ between u and v if $u = 2t - 1$ and $v = t^2$. (6 marks)
- b) Show that a square matrix $A_{n \times n}$ is diagonalizable if and only if A has n linearly independent eigenvectors. (7 marks)

Question Seven (13 Marks)

- a) Consider the bases of \mathbb{R}^3 ; $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $B' = \{(1,1,1), (1,1,0), (1,0,0)\}$.
 i) Find the transition matrix P from B' to B .
 ii) Find the transition matrix Q from B to B'
 iii) Verify that $Q = P^{-1}$
 iv) Compute coordinate matrix of w with respect to B' where $w = (-5, 8, 5)$. (9 marks)
- b) Let $T: V \rightarrow V$ be a linear transformation. Let A be matrix of T with respect to B and A' be matrix of T with respect to B' . Show that $A' = P^{-1}AP$ where P is transition matrix from B' to B . (4 marks)
