ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge...
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAT 212

COURSE TITLE: LINEAR ALGEBRA I

DATE: 31ST JANNUARY, 2022 TIME: 1400 – 1700 HRS

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

REGULAR - MAIN EXAM

MAT 212: LINEAR ALGEBRA I

STREAM: EDS& EDA DURATION: 3 Hours

INSTRUCTIONS
ATTEMPT ALL QUESTIONS IN SECTION A
ATTEMPT ANY THREE QUESTIONS IN SECTION B
DO NOT WRITE ANYTHING ON THIS QUESTION PAPER

SECTION A (31 MARKS): ATTEMPT ALL QUESTIONS

Question One (15 Marks)

(a) Determine the row-rank of the following matrix (4 Marks)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Calculate the adjoint of (5 Marks)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

(c) Find a vector expression for the line through $(6, 1, -3)$ and $(2, 4, 5)$. (3 Marks)

(d) Let $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ be defined by $T((x, y, z)^T) = (x + y - z, x + z)^T$. Determine $[T]$. (3 Marks)

Question Two (16 Marks)

(a) Compute the determinant of (3 Marks)

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 2 & 0 & 7 \\ 1 & -3 & -2 \end{bmatrix}$$

(b) Find the distance from the point $(-1, 2, 1)$ to the line $(1, 1, 1) + t(2, 3, -1)$ (4 Marks)

(c) Solve the following linear system by Gauss elimination method (5 Marks)

$$x + y + z = 3$$
$$x + 2y + 2z = 5$$
$$3x + 4y + 4z = 12$$

(d) Find an equation for the plane perpendicular to $(1, 2, 3)$ and containing the point $(5, 0, 7)$ (4 Marks)

SECTION B (39 MARKS): ATTEMPT ANY THREE QUESTIONS

Question Three (13 Marks)

(a) Find the intersection of the planes $\pi_1 : 2x - y + z = 3$ and $\pi_2 : x + 2y + 3z = 0$. (7 Marks)

(b) Let $S = \{(1, 1, 1, 1), (1, 1, -1, 1), (1, 1, 0, 1), (1, -1, 1, 1)\}$ be a subset of $\mathbb{R}^4$. Find a basis of $L(S)$ (6 Marks)

Question Four (13 Marks)

(a) Determine the range and null space of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ with $T(x, y, z) = (x - y + z, y - z, x, 2x - 5y + 5z)$ (10 Marks)

(b) Consider $V = \mathbb{R}^2$ and for $a \in \mathbb{R}$, $U_a = \{(x, y) \in \mathbb{R}^2 : x + y = a\}$. When is U_a a linear subspace? (3 Marks)

Question Five (13 Marks)

(a) Is $(1, 1, 3)$ a linear combination of $(-1, 2, 1)$ and $(1, 3, 1)$? (6 Marks)

(b) Solve the following simultaneous system using Cramer's rule (7 Marks)

$$x + y + z = 4$$
$$2x - 3y + 4z = 33$$
$$3x - 2y - 2z = 2$$

Question Six (13 Marks)

Find the matrix of cofactors and that of minors of the following matrix

$$A = \begin{bmatrix} 4 & -7 & 6 \\ -2 & 4 & 0 \\ 5 & 7 & -4 \end{bmatrix}$$

Question Seven (13 Marks)

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

using Gauss-Jordan method. (10 Marks)

(b) Find the distance from the point $(1, 2, 3)$ to the plane $2x - y + 3z = 5$. (3 Marks)