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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2021 /2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS

COURSE CODE: STA 424

COURSE TITLE: STOCHASTIC PROCESSES

DATE: 2ND JUNE, 2022 TIME: 9AM – 12 NOON

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES PLEASE TURN OVER

REGULAR – MAIN EXAM

STA 424: STOCHASTIC PROCESSES

STREAM: BSC (Applied Statistics) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. *Answer ALL questions from section A and ANY THREE Questions in section B.*

ii. *All questions in section B carry Equal Marks.*

iii. *Do not write on the question paper.*

SECTION A (31 marks): Answer ALL questions

QUESTION ONE (16MKS)

a) Define the following terms

i Stochastic process (2 Marks)

ii Discrete – time (2 Marks)

iii State space, S (2 Marks)

iv Strict-sense stationary (SSS) (2 Marks)

b) Let N(t) be a Poisson process with intensity $\lambda=2$, and let $X_1, X_2, \cdots\cdots$ be the corresponding arrival times. Find the probability that the first arrival occurs after t=0.5, i.e., $P(X_1>0.5)$

(2 Marks)

c) $\{a_k\} = \{0,0,0,1,1,1\ldots\}$ Find A(S) (2 Marks)

d) Consider the Markov chain with three states, S={1,2,3}, that has the following transition

matrix $p=\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

If we know $P(X_1=1)=P(X_1=2)=\frac{1}{4}$, find $P(X_1=3, X_2=2, X_3=1)$. (4 Marks)

QUESTION TWO (15 Marks)

a) Let X have a Binomial distribution such that $P_k = \binom{n}{k} p^k q^{n-k}$ k = 0,1,2.... Find the probability generating function (p.g,f), mean and the variance (6 Marks)

b) Let X have a Poisson distribution such that $P_k = \frac{\lambda^k e^{-\lambda}}{k!}$ k = 0,1,2.... Find the probability generating function (p.g,f), mean and the variance (6 Marks)

c) $\{a_k\} = \left\{\frac{1}{k!}\right\} \quad \forall k$ Find A(S) (3 Marks)

SECTION B (39 MARKS, CHOOSE ANY THREE QUESTIONS)

QUESTION THREE (13 MARKS)

a) Let $X_1, X_2, \ldots$ be independent Bernoulli r.v.'s with $P(X_n = 1) = p$ and $P(X_n = 0) = q = 1\text{-}p$ for all n. The collection of r.v.'s $\{X_n, \ n \geq 1\}$ is a random process, and it called a Bernoulli process.

(i) Describe the Bernoulli process. (1 Marks)

(ii) Construct a typical sample sequence of the Bernoulli process. (2Marks)

(iii) Find the transition matrix of this process considering a sequence of coin flips, where each flip has probability of having the same outcome as the previous coin flip, regardless of all previous flips (2 Marks)

b) The number of customers arriving at a grocery store can be modelled by a Poisson process with intensity λ=10 customers per hour.

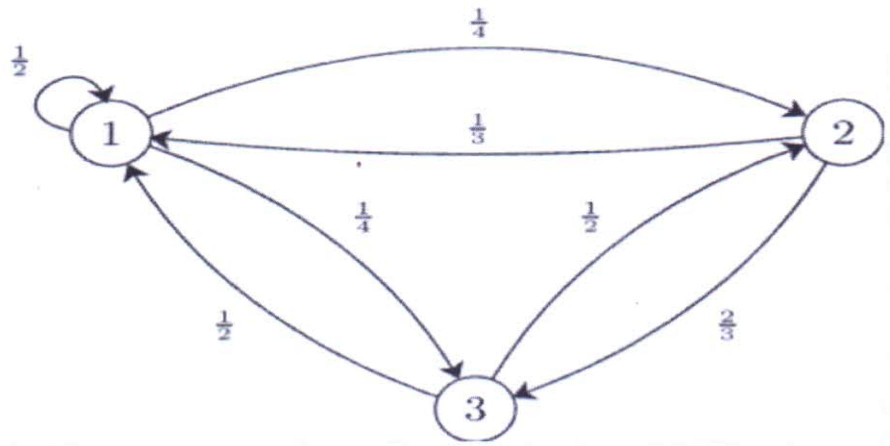
i. Find the probability that there are 2 customers between 10:00 and 10:20. (4 Marks)

ii. Find the probability that there are 3 customers between 10:00 and 10:20 and 7 customers between 10:20 and 11. (4 Marks)

QUESTION FOUR (13 Marks)

a) Let $\{a_k\} = \{1,1,2,3,5,8,13,21, \ldots.\}$ Be a Fibonacci series where $a_0 = 1, a_1 = 1,$ $a_k = a_{k-1} + a_{k-2}$. Find a_{25} (7 Marks)

b) Consider the Markov chain shown below



i. Is this chain irreducible? (1 Marks)

ii. Is this chain aperiodic? (1 Marks)

iii. Find the stationary distribution for this chain. (2 Marks)

iv. Is the stationary distribution a limiting distribution for the chain? (2 Marks)

QUESTION FIVE (13 Marks)

a) $a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0,$ $n \geq 3$ .Subject to initial value $a_0 = 0$ $a_1 = 1$ and $a_2 = -1$ Find A(s) and thus the value of a_k (7 Marks)

b) $a_n = 5_{n-1} - 6a_{n-2}$ $n \geq 2$ .Subject to initial value $a_0 = 1$ $a_1 = 2$. Find A(s) and thus the value of a_k (6 Marks)

QUESTION SIX (13 Marks)

a) Student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. Find

(i) The transition matrix of this process. (2 Marks)

(ii) The transition matrix after 4 nights. (1 Marks)

(iii) Suppose that on the first night, the student toss affair die and studied if only if a 2 or 3 are appeared. What is the probability that he didn't study in the fourth night. (2 Marks)

b) Suppose the process $\{X_{t},: t \geq 0\}$ be a poisson process having rate λ=2 . Find:

i. $P\{X_2 = 4, X_5 = 12, X_9 = 16\}$ (4 Marks)

ii. $P\{X_{1.5} = 10, X_{3.5} = 18, X_5 = 20\}$ (4 Marks)

QUESTION SEVEN (13 Marks)

a) Let $\{Z_0, Z_1, Z_2, Z_3, ...\}$ be a branching process with $Z_0 = 1$. Let Y denote the family size distribution, and suppose that $E(Y) = \mu$. Prove that $E(Z_n) = \mu^n$ (7 Marks)

b) A salesman's area consists of three cities, A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. however, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city. Find

(i) The transition matrix of this process. (3 Marks)

(ii) In the long run, how often does he sell in each of the cities. (3 Marks)

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