Mathematical Model of Variable Speed Pitch Regulated Turbine in a Wind Energy Conversion System

Alfred Wanyama Manyonge
Maseno University

Jacob Otieno Rading
The Catholic University of Eastern Africa

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MATHEMATICAL MODEL OF VARIABLE SPEED PITCH REGULATED TURBINE IN A WIND ENERGY CONVERSION SYSTEM

K. Korkoren[1], A. W. Manyonge[2] and J. Rading[3]

[1]*University of Eldoret, P. O. Box 1125, Eldoret-Kenya*
[2]*Centre for Research on New and Renewable Energies, Maseno University, P. O. BOX 333, Maseno-Kenya*
[3]*The Catholic University of Eastern Africa, P. O. BOX 62157, Nairobi-Kenya*
E-mails: korkorenkenneth@gmail.com; wmanyonge@gmail.com; radingj@yahoo.com

ABSTRACT: *Pitch control of wind turbine in a wind energy conversion system involves the change of the rotor blade pitch angle along the rotor axis.Most compa-nies are faced with the challenge of maintaining a steady and optimal power supply. The control involves the pitch angle of the blades, wind speed and the angular velocity of the turbine. The main objective is to optimize wind energy capture. This is done by a close-loop control through modelling and simulation undertaken and implemented in MATLAB/SIMULINK simulation environment using the power coeficient equation which relates the pitch an-gle and the power output to optimize energy capture. Results of the variable speed and pitch control at steady tip speed ratio are obtained graphically. In conclusion,the simulation shows that only small changes of pitch angle are re-quired to maintain the power output at rated power.*

Keywords: *Pitch angle,Wind velocity, Turbine power, Power coeficient, Tip speed ratio, Generator, Grid*

1. INTRODUCTION

Wind energy model system can be subdivided into the following components: (i) model of the wind, (ii) turbine model, (iii) shaft and gearbox model, (iv) generator model and (v) control system model.There are two types of design models for wind turbines which are classified on the basis of their axis in which the turbines rotate: Horizontal Axis Wind Turbine (HAWT) and Vertical Axis Wind Turbine (VAWT). The VAWT is also called Darrieus rotor named after its inventor [1] . HAWT have the ability to collect maximum amount of wind energy for time of day and season and their blades can be adjusted to avoid high wind storm. Wind turbines operate in two modes namely constant and variable speed. For a constant speed turbine, it produces less energy at low wind speeds than does a variable wind speed turbine which is designed to operate at a rotor speed proportional to the wind speed below its rated wind speed [2] which its units are usually equipped with a blade pitching system [3].A typical wind energy conversion system (WECS) consists of three major devices making up a wind turbine that convert wind energy to electric energy i.e the rotor,nacelle and the tower [4].

1.1. Control System Model

Some of the main objectives of a wind turbine control system as outlined by [5] are Energy capture and Power quality. The control techniques used in wind turbines are pitch control, yaw control and stall control [8]. The output power or torque of a wind turbine is determined by several factors. Among them are (i) angular velocity of turbine, (ii) rotor blade tilt, (iii) pitch angle of the rotor blades (iv) size and shape of turbine, (v) area of turbine, (vi) rotor geometry whether it is a HAWT or a VAWT, (vii) and wind speed. A relationship between the output power and the various variables constitute the mathematical model of the wind turbine. A mathematical model of wind turbine is essential in the understanding of the behaviour of the wind turbine over its region of operation and also modelling enables control of wind turbines performance.

A wind turbine is always designed for specific rated conditions and is designed to generate rated power which is the maximum power that the generater is designed to deliver. Our interest is on the above rated (Blade Pitch Control) where we vary rotor speed and pitch angle to regulate load and maintain power production at rated value.

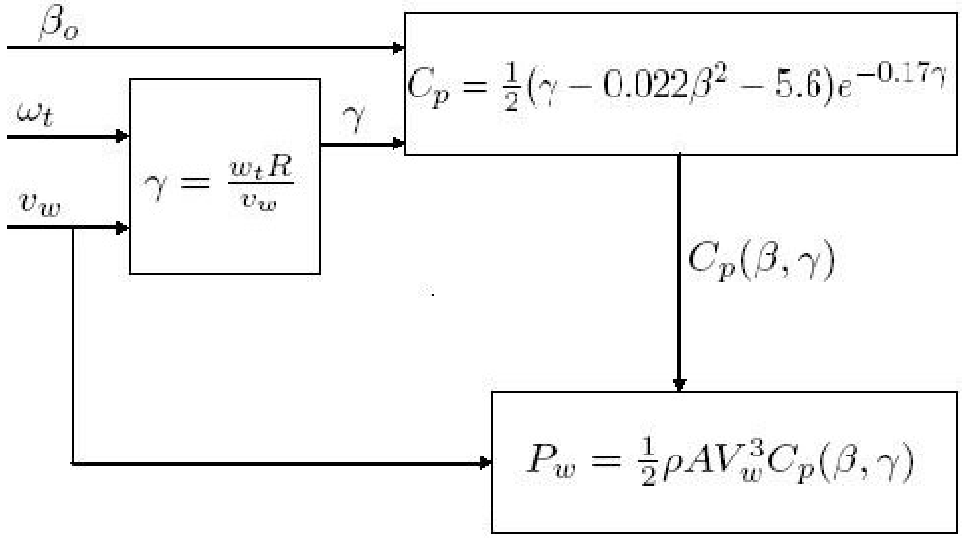

Figure 1.1: Control System Model

1.2. Mathematical Formulation of Control System Model

The kinetic energy in air of mass m moving with speed V is given by;

$$\text{Kinetic Energy} = \frac{1}{2}.m.V^2 \tag{1.2.1}$$

The power in moving air is the flow rate of kinetic energy per second. There-fore:

$$P = \frac{1}{2}(\rho.A.V.).V^2 = \frac{1}{2}\rho.A.V^3 \tag{1.2.2}$$

The actual power extracted by the rotor blades is the difference between the upstream and the downstream wind powers. That is, using Equation (1.3.3):

$$P_o = \frac{1}{2}.mass\ flow\,rate.[V^2 - V_o^2] \tag{1.2.3}$$

where; P_o = mechanical power extracted by the rotor

V = upstream wind velocity at the entrance of the rotor blades

V_o = downstream wind velocity at the exit of the rotor blades

Mass flow rate of air through the rotating blades is derived by multiplying the density with the average velocity. i.e

$$mass\ flow\,rate = \rho.A.\frac{V + V_o}{2} \tag{1.2.4}$$

Replacing equation 1.3.5 into equation 1.3.4 gives;

$$P_o = \frac{1}{2}[\rho.A.\frac{V + V_o}{2}](V^2 - V_o^2) \tag{1.2.5}$$

The above expression can be algebraically rearranged:

$$P_o = \frac{1}{2}\rho.A.V^3 \frac{\left(1 + \frac{V_o}{V}\right)\left[1 - \left(\frac{V_o}{V}\right)^2\right]}{2} \tag{1.2.6}$$

The power extracted by the blades is expressed as a fraction of the upstream wind power as follows:

$$P_o = \frac{1}{2}\rho.A.V^3.C_p \tag{1.2.7}$$

where;

$$C_p = \frac{\left(1 + \frac{V_o}{V}\right)\left[1 - \left(\frac{V_o}{V}\right)^2\right]}{2} \tag{1.2.8}$$

Equation 1.3.9 shows that the turbine captures only a fraction of this power. The power captured by the turbine (P_o) can be expressed as [6],

$$P_o = P_w \times C_p \tag{1.2.9}$$

Where C_p is a fraction called the power coefficient. The power coefficient represents a fraction of the power in the wind captured by the turbine and has a theoretical maximum of 0.55 [7]. The power coefficient can be expressed by a typical empirical formula as

$$C_p = \frac{1}{2}(\gamma - 0.022\beta^2 - 5.6)e^{-0.17\gamma} \tag{1.2.10}$$

where β is the pitch angle of the blade in degrees and γ is the tip speed ratio of the turbine, defined as

$$\gamma = \frac{v_w(mph))}{w_b(rads^{-1})} \tag{1.2.11}$$

where, w_b -Turbine angular speed.

As seen in the power equation, the output power of the wind turbine varies linearly with the rotor swept area. For the horizontal axis turbine, the rotor swept area is given by:

$$A = \frac{\pi}{4}D^2 \tag{1.2.12}$$

where D is the rotor diameter.

For the Darrieus vertical axis machine, determination of the swept area is complex, as it involves elliptical integrals. However, approximating the blade shape as a parabola leads to the following simple expression for the swept area [1]:

$$A = \frac{2}{3}\ \{maximum\ rotor\ width\ of\ the\ centre\}\{height\ of\ the\ rotor\} \tag{1.2.13}$$

The wind power varies linearly with the air density sweeping the blades. The air density ρ varies with pressure and temperature in accordance with the gas law:

$$\rho = \frac{p}{R.T} \tag{1.2.14}$$

where p = air pressure

T = temperature on the absolute blade

R = gas constant

The air density at sea level, one atmospheric pressure (14.7 psi) and 60 ^oF is 1.225kg/m^3. Using this as the reference, ρ is corrected for the site specific temperature and pressure. The temperature and the pressure both in turn vary with the altitude. Their combined effect on the air density is given by the following equation, which is valid up to 6,000 meters (20,000 feet) of site elevation above the sea level:

$$\rho = \rho_o e^{-\left(\frac{0.297 H_m}{3048}\right)} \tag{1.2.15}$$

The air density correction at high elevations can be significant. For example, the air density at 2,000-meter elevation would be 0.986kg/m^3, 20 per cent lower than the 1.225kg/m^3 value at sea level. For ready reference, the temperature varies with the elevation [1]:

$$T = 15.5 - \left(\frac{19.83 H_m}{3048}\right)^o C \tag{1.2.16}$$

1.3. Modelling and Control of the Power Output

One concept that is fundamental to the control dynamics is that the speed change is relatively slow because of the large inertia involved. Pitch con-trol can be better used to regulate power flow especially when near the high speed limit.The generator output can be controlled to follow the commanded power.From equation (1.2.9) it can be shown that;

$$C_p = \frac{P_o}{P_w}$$

where $P_o = P_{target}$

TSR is the ratio between the linear speed of the tip of the blade with respect to the wind speed. The power coefficient C_p varies with the tip-speed ratio, and TSR is given by the equation;

$$\gamma = \frac{w_t R}{V} \tag{1.3.2}$$

Hence,

$$\omega_t = \frac{\gamma_{opt}}{R} V \tag{1.3.3}$$

From eqn (1.3.3), the power production from the wind turbine can be maxi-mized if the system is operated at maximum C_p. As the wind speed changes, the rotor speed should be adjusted to follow the change. This is possible with a variable-speed wind turbine. Wind speed cannot be reliably measured thus to avoid using the wind speed, the equation to compute the target power can be rewritten by substituting the wind speed V and the C_p in the Power extracted from the wind given by;

$$P_w = \frac{1}{2} \rho A V_w^3 C_p(\beta, \gamma) \tag{1.3.4}$$

The target power P_{target} can then be written as;

$$P_{target} = 0.5 \rho A C_{p-target} \left[\frac{R}{\gamma_{target}} \right]^3 w_t^3 \tag{1.3.5}$$

It can be seen that the P_{target} is proportional to the cube of the rotor speed. To prevent rotor speed from becoming too high, the extracted power from incoming wind must be limited. This can be done by reducing the coefficient of performance of the turbine (the C_p value).

Since $C_{p-target} = C_{p-max}$ = constant,in this region, from (1.3.3):

$$P_w = KV^3 \tag{1.3.6}$$

where;

$$K = \frac{1}{2} \rho A C_{p-\max}(\beta, \gamma) = constant \tag{1.3.7}$$

The wind speed is varied, turbine speed is maintained at rated speed $w_t = w_{t-rated}$ and corresponding γ is calculated using (1.3.2). The power output is maintained at rated power ($P_{rated} = P_{target}$). The C_p corresponding to rated power is calculated using:

$$C_p = \frac{P_{target}}{0.5 \rho A V_w} \tag{1.3.8}$$

Therefore,

$$\beta = \sqrt{\left(\frac{1}{0.022} \left(\gamma - 5.6 - \frac{2 P_{target} e^{-0.17\gamma}}{P_w} \right) \right)} \tag{1.3.9}$$

Here pitch control is used to control the output power above their rated wind speed. Next, MATLAB is used to simulate and perform an analysis of the variation of C_p against γ since $C_p = f\{\gamma, \beta\}$, the plot of C_p vs

γ at various values of β is shown in Figure 1.2. With varying β, we obtain curves called performance curves for a given best turbine and simulate its best operation range. From Figure 1.2, in normal operation, pitch angle control with rotational speeds are expected at $\beta_n = 5^0$ to $\beta_n = 10^0$. Here its clear that there is a reduction of C_p from 38% when the angle is 0 to about 28% when the angle is 10^0. The simulation also shows that the maximum rotor efficiency C_p is achieved at a particular TSR, which is specific to the aerodynamic design of a given turbine. For this case the TSR needed for the maximum power extraction is approximately, $\gamma = 13$.

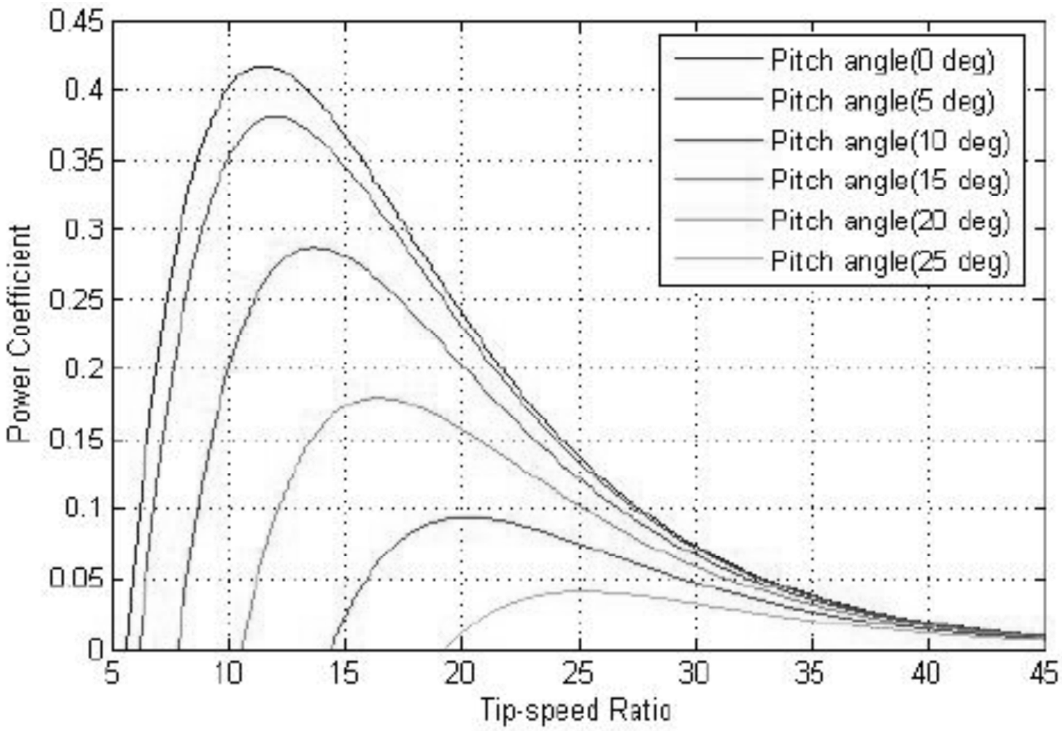

Figure 1.2: Power Coefficient vs. tip Speed Ratio for Various Values of Pitch Angles

In conclusion,simulation shows that the wind turbine can be operated at its optimum energy capture while minimizing the load on the wind turbine for a wide range of wind speeds and it is operated at high C_p values most of the time. To prevent rotor speed from becoming too high, the extracted power from incoming wind must be limited. This can be done by reducing the coefficient of performance of the turbine (the C_p value). The C_p value is manipulated by changing the pitch angle (see Figure 1.2). The blades are considerably heavy in a large turbine, therefore, the rotation must be facilitated by either hydraulic or electric drives.Since target power can be controlled instantaneously at any angular speed of the turbine and the upper limit is the only rating of the power converter and generator, it shows that only small changes of pitch angle are required to maintain the power output at rated power thus a higher pitch rate capability of a wind turbine can lessen the requirement for the generator and power converter rating.

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