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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MAIN EXAM

COURSE CODE: MAT 113

COURSE TITLE: DIFFERENTIAL CALCULUS

DATE: 16/02/2021 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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MAT 113
REGULAR - MAIN EXAM

MAT 113: DIFFERENTIAL CALCULUS

STREAM: BSC (CS & ACS) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. Answer **All** questions from **Section A** and any **Three** from **Section B**

ii. Do not write on the question paper.

SECTION A (31 MARKS)

Question One (16 Marks)

(a) Find the equation of the tangent line to the curve $y = x^4$ at the point $(-1, 1)$. (3 Marks)

(b) Find $\frac{dy}{dx}$ given the curve with parametric equations $y = 4 - t^2$ and $x = t^3 - t$. (3 Marks)

(c) Estimate the value of the following limit $\lim_{x \to 2} \frac{x^2+4x-12}{x^2-2x}$. (3 Marks)

(d) Let $g(x) = 3(4 - 9x)^4$ find $g'(x)$. (2 Marks)

(e) Find the slope of the tangent to the curve $x^2 + x + y^2 = 20$ at the point $(1, 2)$. (2 Marks)

(f) Find the linear approximation of $h(t) = t^4 - 6t^3 + 3t - 7$ at $t = -3$. (3 Marks)

Question Two (15 Marks)

(a) Define the derivative of a function from first principles and hence find $f'(x) = \sqrt{x}$. (4 Marks)

(b) Consider the functions $f(x) = x^3 + x$ and $g(x) = x^2 + 3$. Compute $f \circ g$. (2 Marks)

(c) Find the vertical asymptote(s) of the function $f(x) = \frac{2x-3}{x+1}$. (3 Marks)

(d) Determine whether or not the function $f(x)$ is continuous at the point $x = 1$, where

$$f(x) = \begin{cases} x^2, & \text{for } x \leq 1 \\ 2 - x, & \text{for } x > 1. \end{cases}$$ (3 Marks)

(e) Use L'Hospital's rule to evaluate $\lim_{x \to 0} \frac{e^{3x}-1}{x^2}$. (3 Marks)

SECTION B (39 MARKS)

Question Three (13 Marks)

(a) Consider the parametric curve $x(t) = -2 + 2\cos t$ and $y(t) = 1 - 2\sin t$. Find the point of the curve for which the normal line to the curve has a slope of -1. (3 Marks)

(b) Verify the mean value theorem for $f(x) = x^3 - 3x^2 - 10x + 20$, on the interval $[-1, 5]$. (4 Marks)

(c) Determine whether the function $f(x) = \frac{4x+5}{9-3x}$ is continuous at the point $x = 3$. (3 Marks)

(d) Find the limit $\lim_{x \to \infty} \frac{x-1}{2x^2+3}$ (3 Marks)

Question Four (13 Marks)

(a) Determine where the given function is discontinuous $g(x) = \frac{x^2-1}{3x^2+2x-8}$. (3 Marks)

(b) Find $\frac{dy}{dx}$ of the function $y^2 = x^2 + \sin xy$. (4 Marks)

(c) A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \tan^{-1}(t)$. Find the velocity of the particle when $t = 16$. (3 Marks)

(d) Find $\lim_{x \to 1} \frac{x^2-1}{x-1}$. (3 Marks)

Question Five (13 Marks)

(a) Find the equation of the normal line to the curve $y = x^3 + e^x$ at $x = 0$. (4 Marks)

(b) The profit of XYZ, a manufacturing concern, is defined by the function $P = 5000 + 200x - 2x^2$, where x is the amount of raw materials needed to produce commodity A. Determine

(i) the amount of raw material that will maximize the profit. (3 Marks)

(ii) the maximum profit. (3 Marks)

(c) Test for differentiability of the function $f(x) = x - 2$ at $x = 2$. (3 Marks)

Question Six (13 Marks)

(a) Using the definition of monotonicity, prove that the function $f(x) = x^2 + 1$ is strictly increasing for $x \geq 0$. (5 Marks)

(b) Find $\frac{dy}{dx}$ at $x = 4$ if $y = \sqrt{x}(\sqrt{x} + 1)$. (3 Marks)

(c) Differentiate the function $y = \ln[(x^2 + 5)(x^3 - 3)]$. (3 Marks)

(d) Evaluate $\lim_{t \to 0} H(t)$ where

$$H(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } x \geq 0. \end{cases}$$ (2 Marks)

Question Seven (13 Marks)

(a) Define the continuity of a function f at a point a, hence determine whether or not the function below is continuous at the point $x = 2$

$$f(x) = \begin{cases} 1 + x, & \text{if } x \leq -2 \\ 2 - x, & \text{if } -2 < x \leq 2 \\ 2x - 1, & \text{if } 2 < x. \end{cases}$$ (4 Marks)

(b) Wafula needs to enclose a rectangular field with a fence. He has $500ft$ of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area. (5 Marks)

(c) List the members of the following sets

(i) $A = \{x | x \in \mathbb{N}\}$ & x is a multiple of 3. (2 Marks)

(ii) $B = \{x | x \in \mathbb{Z}\}$ & $0 < x \leq 6$. (2 Marks)