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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: **MAT 213**

COURSE TITLE: **LINEAR ALGEBRA II**

DATE: 23/7/2021 **TIME: 0800-1100HRS**

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

REGULAR – MAIN EXAMINATION
MAT 213: LINEAR ALGEBRA II

STREAM: ASC **TIME: 3 HRS**
EXAMINATION SESSION: JULY **YEAR: 2020/2021**

INSTRUCTIONS TO CANDIDATES

(i) Answer all questions in section A (Compulsory)
(ii) Answer any other THREE questions in section B
(iii) Answers should be comprehensive, informative and neat.

SECTION A (31 MARKS)

Question One (16 Marks)

a). Define the following terms

i). A spanning set of a vector space, V **(1 Mark)**

ii). An eigenvector of a matrix M **(1 Mark)**

iii). A quadratic form **(2 Marks)**

iv). A symmetric matrix **(1 Marks)**

b). Prove that set of symmetric matrices is a subspace of $n \times n$ square matrices **(4 Marks)**

c). Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$. **(7 Marks)**

Question Two (15 Marks)

a). Determine the inverse of the matrix Q that diagonalizes matrix $B = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$. **(6 Marks)**

b). Show that the matrix $A = \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}$ is orthogonal. **(3 Marks)**

c). (i). Find a quadratic form corresponding to the following symmetric matrix

$$B = \begin{bmatrix} 4 & -4 & 9 \\ -4 & 9 & -3 \\ 9 & -3 & 1 \end{bmatrix}.$$ **(2 Marks)**

(ii). Classify the matrix as either positive definite, negative definite or indefinite. Show your working. **(4 Marks)**

SECTION B (39 MARKS)

Question Three (13 Marks)

a). Prove that the orthogonal matrices are isometric. **(3 Marks)**

b). Consider the bases $B = \{(1,2),(3,4)\}$ and $C = \{(7,3),(4,2)\}$ of $\mathbb{R}^2$. Let $v = (1,0)$. Find

(i). The coordinate of v with respect to B, $[v]_B$ and C, $[v]_C$. **(4 Marks)**

(ii). The change of basis matrix from C to B, $P_{B\leftarrow C}$ and use it to determine the change of basis matric from B to C, $P_{C\leftarrow B}$. **(6 Marks)**

Question Four (13 Marks)

Orthogonally diagonalize matrix A, $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}$. **(13 Marks)**

Question Five (13 Marks)

(a). Define the term inner product space giving an example **(4 Marks)**

b). Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\langle -3,2\rangle) = \langle -4,2\rangle$ and $T(\langle 3,-5\rangle) = \langle -11,4\rangle$. Find the matrix A such that $T(x) = Ax$. **(5 Marks)**

c). Show that the vector $v = (4,1,-2), u = (-3,0,1)$ and $w = (1,-2,1)$ are linearly independent. **(4 Marks)**

Question Six (13 Marks)

a). Prove that an orthogonally diagonalizable matrix must be symmetric. **(5 Marks)**

b). Differentiate between algebraic and geometric multiplicity of an eigenvalue of a matrix M. **(2 Marks)**

c). Find the minimum polynomial of the matrix $A = \begin{pmatrix} 4 & 0 & -3 \\ 4 & -2 & -2 \\ 4 & 0 & -4 \end{pmatrix}$. **(6 Marks)**

Question Seven (13 Marks)

Find a change of variable that will reduce the quadratic form $Q(x_1, x_2, x_3) = x_1^2 - x_3^2 - 4x_1x_2 - 4x_2x_3$ to the sum of squares and express the quadratic form in terms of the new variables. **(13 Marks)**

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