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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: STA 421

COURSE TITLE: STATISTICAL COMPUTING

DATE: 10/03/2021 **TIME:** 1400 – 1700 HRS

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES **PLEASE TURN OVER**

REGULAR – MAIN EXAM

STA 421: STATISTICAL COMPUTING

STREAM: **DURATION: 3 hours**

INSTRUCTION TO CANDIDATES

Answer **ALL** questions from section A and any **THREE** from section B.

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SECTION A [31 Marks] Answer All questions]

QUESTION ONE [15 Marks]

a) Define clearly the following terms [4 Marks]
   i) Simulation.
   ii) Pseudo random number generator.
   iii) System
   iv) State of a system

b) Distinguish between deterministic and stochastic simulation models and give an example for each. [4 Marks]

c) State two reasons why simulation is important [2 Marks]

d) Give three situations where simulation has been used. [3 Marks]

e) Identify two properties that random number generators should possess. [2 Marks]

QUESTION TWO [16 Marks]

a) What is the difference between dynamic and static model? [2 Marks]

b) Describe Monte Carlo Simulation [3 Marks]

c) Give a diagrammatic representation of model development life cycle. [3 Marks]

d) State any three random number generators tests. [3 Marks]

e) Why do we build models (as opposed to experiment on actual systems)? [2 Marks]

f) Give three fundamental steps that are used in model building? [3 Marks]

ECTION B [39 Marks] Answer any THREE questions]

QUESTION THREE [13 Marks]

a) Discuss any two classifications of a system. [4 Marks]

b) Using each of the following methods of pseudo random number generators, generate six random numbers;

i) Midsquare method, given $Z_0 = 7182$ [3 Marks]

ii) Linear congruential method, given $Z_0 = 7, a = 5, c = 3, m = 16$ [3 Marks]

iii) Additive congruential method, given $Z_0 = 6, a = 2, c = 5, m = 14$ and $k = 1$ [3 Marks]

QUESTION FOUR [13 Marks]

a) Consider a random variable X which takes on values 1, 2, 3 and 4 with probability 0.15, 0.20, 0.25 and 0.40 respectively. Determine the mean and variance of X. Sketch the probability density function (pdf) and probability distribution function (PDF) of X. [7 Marks]

b) Let X be a random variable and f be a function such that $f(X) \in \Re$, then Monte Carlo estimate for $E(f(X))$ is given by $Z_N^{MC} = \frac{1}{N}\sum_{j=1}^{N} f(X_j)$ where $X_1, X_2, \ldots, X_N$ are i.i.d with the same distribution of X. Write a program that computes Monte Carlo estimates. [6 Marks]

QUESTION FIVE [13 Marks]

a) i) Given the model $y = \beta_0 + \beta_1 x + ey = \hat{\beta}_0 + \hat{\beta}_1 x$. Write down an expression for the error of estimation and describe what it means when it is equivalent to zero and when it gets larger. [4 Marks]

ii) Consider the model $y_i = \beta_0 + \beta_1 x_i + e_i$, $x_i \sim N(4, 0.1)$, $e_i = N(0, 0.5)$, $\beta_0 = 2.5$, $\beta_1 = 1.8$ Write a program in R that generates 10000 variates. [4 Marks]

b) Using the following data for X and Y respectively: (1 , 4), (2 , -1), (1.5 , 3), (-2 , 5), (3 , 2), write down that are used to fit a simple linear regression model and determine the SSE using the for the loop. [5 marks]

QUESTION SIX [13 Marks]

a) Assume $X \sim Exp(1)$ and $Y \sim N(0, X)$, that is Y is normally distributed with a random variance. Use Monte Carlo estimation to estimate $E(X/Y) = 4$ and $Var(X/Y) = 4$ [5 Marks]

b) Assume the following density unction $f(x) = \begin{cases} 1/x^2 & \text{if } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$

Write a program which uses the inverse transform method to generate random numbers

Test your program and write down code to plot a histogram of 10 000 random numbers together with the density f [8 Marks]

QUESTION SEVEN [13 Marks]

a) A computer repair person is 'beeped' each time there is a call for service. Then number of beep per hour is distributed according to Poisson with $\lambda=2$ per hour. Find the probability of three beeps in the next one hour and two or more beeps in one hour. [5 Marks]

b) Identify and describe four properties of a good arithmetic random number generator. [8 Marks]