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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2020 /2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS

MAIN EXAM

COURSE CODE: MAT 210

COURSE TITLE: CALCULUS II

DATE: 18/03/2021 **TIME: 1400 – 1700 HRS**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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MAT 210

REGULAR - MAIN EXAM

MAT 210: CALCULUS II

STREAM: BSC (CS & ACS) **DURATION: 3 Hours**

INSTRUCTIONS TO CANDIDATES

i. Answer **All** questions from **Section A** and any **Three** from **Section B**

ii. Do not write on the question paper.

SECTION A (31 MARKS)

Question One (16 Marks)

(a) State the Rolle's theorem. (2 Marks)

(b) Solve the integral $\int \cos 5\theta \cos 7\theta d\theta$. (3 Marks)

(c) Use the trapezoidal rule to estimate the area $\int_0^3 (x^2 + x)dx$ by taking 6 equal intervals and compare the result with the exact value of the integral. (5 Marks)

(d) Evaluate the integral $\int_0^3 e^{2x} dx$. (2 Marks)

(e) Find the area enclosed by the curves $y = x^2$ and $y = x^3$. (4 Marks)

Question Two (15 Marks)

(a) Find the volume of the solid generated by revolving about the y-axis the region bounded by the curve $y = 1 + x$ between the lines $y = 2$ and $y = 4$ and about y-axis. (5 Marks)

(b) Find the length of the curve $x = 5(2t - \sin 2t)$, $y = 10 \sin^2 t$ between $t = 0$ and $t = \pi$. (5 Marks)

(c) Define the term indefinite integral. (1 Mark)

(d) Evaluate $\int \frac{x^2+x+1}{2x^3+3x^2+6x+5} dx$. (4 Marks)

SECTION B (39 MARKS)

Question Three (13 Marks)

(a) Derive the reduction formula $I_n = \int x^n e^x dx = x^n e^n - nI_{n-1}$ and use it to evaluate $\int x^3 e^x dx$. (6 Marks)

(b) Use change of variables $t = \tan\frac{x}{2}$ to show that $\sec x dx = \ln\left(\frac{1+t}{1-t}\right) + c$. (7 Marks)

Question Four (13 Marks)

(a) A sphere is generated by revolving the graph of $y^2 = r^2 + x^2$ about x-axis. Verify that the volume of the sphere is $\frac{4}{3}\pi r^3$. (4 Marks)

(b) Show that $\ln(1 + x + x^2) = \int \frac{1}{1-x}dx - 3\int \frac{x^2}{1-x^3}dx + c$ where c is a constant. (4 Marks)

(c) Integrate $\int e^{5x}\cos 3x dx$ by parts showing all your workings. (5 Marks)

Question Five (13 Marks)

(a) Approximate the value of $\int_0^2 \frac{1}{x^2+1}dx$ with $n = 4$ using Simpson's rule and compare with the exact value by finding the percentage error. (7 Marks)

(b) Find the total distance travelled by a particle between $t = 0$ and $t = 3$ if the velocity of the particle is given by the function $v(t) = 3t^2 + 2t + 1$. (3 Marks)

(c) Determine whether the integral $\int_{-\infty}^{0} \frac{1}{(x^2-1)^2}dx$ converges or diverges. (3 Marks)

Question Six (13 Marks)

(a) Evaluate using trigonometric substitution the value of $\int \frac{2}{3+4x^2}dx$. (4 Marks)

(b) Find the area enclosed by the curve with parametric equations $x = a\cos^2\theta\sin\theta$ and $y = a\cos\theta\sin^2\theta$ between $\theta = 0$ and $\theta = \frac{\pi}{2}$. (6 Marks)

(c) Find the average value of $f(x) = 3x^2 + 4x + 1$ between $x = -1$ and $x = 2$. (3 Marks)

Question Seven (13 Marks)

(a) Show that $\int \cos 6x \cos 4x dx = \frac{1}{20}\sin 10x + \frac{1}{4}\sin 2x + c$. (3 Marks)

(b) Integrate $\int \frac{1}{3+\cos^2 x}dx$. (6 Marks)

(c) Find the length of the cardioid $r = 5(1 + \cos\theta)$ from $\theta = 0$ to $\theta = \pi$. (4 Marks)