ALUPE UNIVERSITY COLLEGE
...Bastion of Knowledge....
P. O.Box 845-50400 Busia(K)
principal@auc.ac.ke
Tel: +254 741 217 185
+254 736 044 469
off Busia-Malaba road

OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE /ARTS

COURSE CODE: MAT 311

COURSE TITLE: REAL ANALYSIS II

DATE: 17TH DECEMBER, 2019 TIME: 2.00 PM – 5.00 PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 4 PRINTED PAGES PLEASE TURN OVER

MAT 311: REAL ANALYSIS II

STREAM: **BED SCI/ARTS** **TIME:** 3 HRS

EXAMINATION SESSION: DECEMEBER **YEAR:** 2019/2020

INSTRUCTIONS TO CANDIDATES

(i) *Answer question ONE and TWO (Compulsory)*
(ii) *Answer any other THREE questions*
(iii) *Answers should be comprehensive, informative and neat.*

Question One (16 Marks)

a). Define the following terms

i). A sequence **(1 Mark)**

ii). Limit superior **(1 Mark)**

iii). A partition on closed set $[a, b]$ **(2 Marks)**

iv). Derivative of a functions f at c **(2 Marks)**

b). Let $\{x_n\}$ and $\{y_n\}$ be sequences which converge to x and y respectively. Prove that the sequence $\{x_n + y_n\}$ converge to $x + y$. **(3 Marks)**

c). Prove that a composite function $g \circ f$ is continuous at a if f is continuous at a and g is continuous at $b = f(a)$. **(3 Marks)**

d). Let $f: [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$$

Show that f is not Riemann integrable. **(4 Marks)**

Question Two (15 Marks)

a). (i). What do you understand by the term an absolutely convergent series? **(1 Mark)**

(ii). Prove that a sequence $\{f_n\}$ of bounded functions on a set $D \subseteq \mathbb{R}^n$ to $\mathbb{R}^m$ converges uniformly on D to a function f if and only if $\|f_n - f\| \to 0$. **(5 Marks)**

b). State and prove the maximum value theorem for functions on the real number set $\mathbb{R}$.

(5 Marks)

c). Find the value of x for which the series $\sum_{n=0}^{\infty}(3x)^n$ converges, hence, state the radius of convergence. **(4 Marks)**

Question Three (13 Marks)

a). State and prove the Cauchy's mean value theorem. **(8 Marks)**

b). Using the Cauchy's mean value theorem, evaluate the limit of the function $h(x) = x \log x$ as $x \to 0$. **(5 Marks)**

Question Four (13 Marks)

a) Let $\sum_{n\in\mathbb{N}} x_n$ be a series of elements of $\mathbb{R}$. Prove that the series converges in $\mathbb{R}$ if and only of for each real no $\epsilon > 0$, there is an $N(\epsilon) \in \mathbb{N}$ such that

$$\left|\sum_{k=n}^{m} x_k\right| < \epsilon \ for\ all\ m \geq n \geq N(\epsilon).$$

(6 Marks)

b) (i). Prove that a Cauchy sequence is bounded. **(4 Marks)**

(ii). Prove that the sequence of functions $f_n(x) = \frac{1}{n}\sin(nx + n)$ converges to $f(x) = 0$, hence, identify the mode of convergence. **(3 Marks)**

Question Five (13 Marks)

a). Differentiate between pointwise and uniform convergent of sequences of functions.

(2 Marks)

b). Let f_n be a sequence of functions defined by $f_n(x) = x^n$ for $n \in \mathbb{N}$ and $x \in \mathbb{R}$. Prove that f_n converges to

$$f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \end{cases}$$

and diverges for $x > 1$. **(6 Marks)**

c). Prove that all monotonic increasing functions are functions of bounded variation. **(5 Marks)**

Question Six (13 Marks)

a) Let f be a function defined as $f(x) = \begin{cases} x+3 & \text{for } x \leq 2 \\ x^2 - 1 & \text{for } x > 2 \end{cases}$. Find $\lim_{x \to 2} f(x)$. **(3 Marks)**

b) Let $\{x_n\}$ be a sequence of real numbers which is monotone increasing, then the sequence converges if and only if it is bounded, in which case, its limit is $\sup\{x_n\}$. **(6 Marks)**

c) Let $V_f[a,b]$ be the total variation of the function f. Prove that if $V_f[a,b] = 0$ if an only if $f(x) = c, c \in \mathbb{R}$. **(4 Marks)**

Question Seven (13 Marks)

a) Differentiate between Lower and Upper Darboux sums **(5 Marks)**

b) Let $f: [a,b] \to \mathbb{R}, a, b \in \mathbb{R}$ be bounded and α an increasing function on $[a,b]$. Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be partitions on $[a,b]$ such that $\mathcal{P}_2$ is finer than $\mathcal{P}_1$. Prove that

$$L(f, \alpha, \mathcal{P}_1) \leq L(f, \alpha, \mathcal{P}_2)$$

where $L(f, \alpha, \mathcal{P}_1)$ and $L(f, \alpha, \mathcal{P}_2)$ are Lower and Darboux sums respectively. **(8 Marks)**

****