



OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION

**FOR THE DEGREE OF BACHELOR OF
EDUCATION SCIENCE**

COURSE CODE: PHY 314

COURSE TITLE: QUANTUM MECHANICS 1

DATE: 10TH DECEMBER 2019

TIME: 2:00PM-5:00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF 4 PRINTED PAGES

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REGULAR-MAIN EXAM**PHY 314: QUANTUM MECHANICS 1****STREAM: BED (Science)****DURATION: 3 Hours****INSTRUCTIONS TO CANDIDATES**

- i. Answer questions **ONE** and **TWO** in **SECTION A** and any other **THREE** questions in **SECTION B**.
- ii. You may need to use the following constants
- Planck's constant $h = 6.625 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$
 - Mass of an electron, $M_e = 9.11 \times 10^{-31} \text{ Kg}$
 - Mass of a proton, $M_p = 1.67 \times 10^{-27} \text{ Kg}$
 - Electronic charge, $e = 1.6 \times 10^{-19} \text{ C}$
 - $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$

SECTION A (28 MARKS)**Question One (14 marks)**

- (a) State the boundary conditions in quantum mechanics (3 Marks)
- (b) Normalize the wave function given by $\Psi(\phi) = Ae^{i\phi}$ in the region $0 \leq \phi \leq 2\pi$ (3 Marks)
- (c) A particle of mass m is trapped in a one dimensional box with a potential described by

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{Otherwise} \end{cases}$$

Solve the Schrödinger equation for this potential (4 Marks)

- (d) (i) What is degeneracy? (1 Mark)
- (ii) State Heisenberg's momentum and position uncertainty principles. (3 Marks)

Question Two (14 Marks)

- (a) State two properties of Schrodinger's equation (2 Marks)
- (b) Prove that eigenvalue of the Hermitian operator is real (5 Marks)
- (c) What is the smallest possible uncertainty in the position of an electron moving with velocity 10^6 m/s ? (3 Marks)
- (d) Consider the wavefunction $\Psi(x) = A(ax - x^2)$ for $0 \leq x \leq a$, normalize the wavefunction (4 Marks)

SECTION B (42 MARKS)**Question Three (14 Marks)**

- (a) Consider the hydrogen atom eigenfunction $\Psi_{432}(r, \theta, \phi)$ what is the
- (i) Total energy of an electron in this state in eV (2 Marks)
 - (ii) Total orbital angular momentum. (2 Marks)
- (b) Discuss the failures of classical mechanics from the photoelectric experiment (6 Marks)
- (c) Explain the meaning of the quantum numbers n, l, m_l (3 Marks)
- (d) Explain the meaning of Zeeman effect. (1 Mark)

Question Four (14 Marks)

- (a) Consider a particle of mass m held in a one-dimensional potential $V(x)$. Suppose that in some region $V(x)$ is constant $V(x) = V$. For this region, find the stationary states of the particle when (i) $E > V$, (ii) $E < V$, and (iii) $E = V$, where E is the energy of the particle. [10 Marks]

- (b) Show that $[x, p_x] = i\hbar$, explain the meaning of these results. (4 Marks)

Question Five (14 marks)

- (a) Calculate the wavelength associated with a particle of mass 2 g moving with velocity of 3.3125 km/s (3 Marks)
- (b) Given the wave equation $\Psi = Ae^{i(kx - \omega t + \Phi)}$, construct the one-dimensional Schrödinger wave equation. (9 Marks)

Question Six (14 marks)

- (a) Write short notes on operators used in quantum mechanics. (4 Marks)
- (b) Consider the wave function; $\Psi(r, t) = [Ae^{-ipx/\hbar} + Be^{ipx/\hbar}]e^{-ip^2t/2m\hbar}$
Find the probability current corresponding to this wave function (10 Marks)

Question Seven (14 Marks)

Consider a particle subjected to a time-independent potential $V(r)$.

- a) Assume that a state of the particle is described by a wave function of the form $\Psi(r, t) = \phi(r)\chi(t)$. Show that $\chi(t) = Ae^{-i\omega t}$ (A is a constant) and that $\phi(r)$ must satisfy the equation $-\frac{\hbar^2}{2m}\nabla^2\phi(r) + V(r)\phi(r) = \hbar\omega\phi(r)$, where m is the mass of the particle. (10 Marks)
- b) Prove that the solution of the Schrödinger equation of part (a) leads to a time-independent probability density. (4 Marks)
