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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION/ BACHELOR OF SCIENCE (APPLIED STATISTICS WITH COMPUTING)

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 27TH OCTOBER, 2020 TIME: 1400 – 1700 HRS

INSTRUCTION TO CANDIDATES

- **SEE INSIDE**

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REGULAR –MAIN EXAM

MAT 214: *VECTOR ANALYSIS*

STREAM: BED/ASC DURATION: 3Hrs

Hours

INSTRUCTION TO CANDIDATES

i. *Answer ALL questions from* ***section A*** *and any* ***THREE*** *from* ***section B***

ii. *Do not write on the question paper.*

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SECTION A (31 MARKS): ANSWER ALL QUESTIONS

SECTION A: 31 MARKS (COMPULSORY SECTION)

QUESTION ONE (16 marks)

a) Define the following terms as used in vectors
   i) Gradient (1 mark)
   ii) Divergence (1 mark)
   iii) Curl. (1 mark)
   iv) Irrotational vector. (1 mark)
   v) Conservative force field. (1 mark)

b) Consider the vectors $\vec{A} = 4\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{B} = \hat{i} + \hat{j} + 3\hat{k}$, find the following
   i) Their magnitudes,
   ii) Their directional cosines,
   iii) Unit vector in the direction of $\vec{A}$ (6 marks)

c) If $\vec{C} = (5xy^2)\hat{i} - \cos(xy)\hat{j} + e^{(x+y)}\hat{k}$, find $\frac{\partial \vec{C}}{\partial x}$, and $\frac{\partial^2 \vec{C}}{\partial x \partial y}$. (2 marks)

d) Find the unit tangent, $\hat{T}$, to any point on the curve $\vec{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}$, hence find the unit tangent at point $t = 2$. (3 marks)

QUESTION TWO (15 marks)

a) Given vector $\vec{K} = (r^3+2)\hat{i} + \hat{j} + 3r^2\hat{k}$, and $\vec{M} = \hat{j}\cos r - \hat{k}\sin r$, find $\frac{d}{dr}(\vec{M} \times \vec{K})$ (2 marks)

b) $\phi = 3x^3z - xy^3$, find $\nabla\phi$ (2 marks)

c) If $\vec{A} = (3x^2 - 6yz)\hat{i} + (2y + 3xz)\hat{j} + (1 - 4xyz^2)\hat{k}$, evaluate $\int_c \vec{A} \cdot d\vec{r}$ from (0,1,1) to (1,1,1) along following the path $x = t$, $y = t^2$, $z = t^3$. (6 marks)

d) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_c xdy - ydx$ using Green's theorem, hence find the area of an ellipse where $x = \cos\theta$ and $y = \sin\theta$. (5 marks)

SECTION B: 39 MARKS (ATTEMPT ANY THREE QUESTIONS)

QUESTION THREE (13 MARKS)

State and prove Green's theorem. (13 marks)

QUESTION FOUR (13 MARKS)

a) Given that $\phi = \cos(xy)$ and $\vec{h} = (4x^2yz)\hat{i} + (y^3z)\hat{j} + (z^2)\hat{k}$, find,

i) $curl\ \vec{h}$, (2 marks)

ii) $\vec{h} \cdot div\ \phi$ (4 marks)

b) A bird is flying creating a path whose curve is given by the parametric equation $x = \cos 5t,\ \ y = e^{2t}$, $z = 3\sin 5t$.

i) Determine its velocity and acceleration at any time, t.

ii) Find the magnitude of the velocity and acceleration at $t = 2$. (4 marks)

c) Show whether the vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{C} = \hat{i} + 2\hat{j} - 2\hat{k}$ can form sides of a right angle triangle. (3 marks)

QUESTION FIVE (13 MARKS)

Given the parametric equations $x = t$, $y = t^2$, and $z = \frac{2}{3}t^3$, find,

i) The curvature

ii) Principal binormal. (13 marks)

QUESTION SIX (13 MARKS)

a) If $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(ds)^2$ in cylindrical co-ordinates (3 marks)

b) State Stokes theorem. (2 marks)

c) Verify Stokes theorem for $\vec{A} = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$ where S is the surface of a paraboloid $2z = x^2 + y^2$. The boundary c of S is a circle with equations $x^2 + y^2 = 4$, $z = 2$ and parametric equations $x = 2\cos t$, $y = 2\sin t$, $z = 2$, where $0 \le t < 2\pi$. (8 marks)

QUESTION SEVEN (13 MARKS)

a) Evaluate $\int_{(0,1)}^{(1,2)} (x^2 - y)dx + (y^2 + x)dy$ along a straight line from $(0,1)$ to $(1,2)$. (5 marks)

b) If $f = (x^3yz^2)\hat{i} + (2y^2z)\hat{j} - (xz)\hat{k}$, find the divergence at $(1,4,2)$. (3 marks)

c) Evaluate $\iint_S \vec{C} \cdot \vec{n} ds$ where vector $\vec{C} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, and S is the surface of the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ using divergence theorem. (5 marks)