

OFFICE OF THE DEPUTY PRINCIPAL ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS 2019 /2020 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF **EDUCATION SCIENCE/ARTS**

COURSE CODE:

MAT 114

COURSE TITLE:

INTEGRAL CALCULUS

DATE: 16TH OCTOBER, 2020 TIME: 0900 – 1200 HRS

INSTRUCTION TO CANDIDATES

SEE INSIDE

THIS PAPER CONSISTS OF 3 PRINTED PAGES

PLEASE TURN OVER

REGULAR - MAIN EXAM

MAT 114: INTEGRAL CALCULUS

STREAM: EDS, EDB, EDA

DURATION: 3 Hour

INSTRUCTION TO CANDIDATES

Answer ALL questions from section A and any THREE from section B.

No sharing of scientific calculators.

Do not write on this question paper.

Duration of the examination: 3 hours

SECTION A (31 MARKS): ANSWER ALL QUESTIONS

QUESTION ONE (16 MARKS)

a) Differentiate between definite and indefinite integrals (2 marks)

- b) Find the values of c which satisfy the mean-value theorem for $f(x) = 3x^2 + 5x 2$ on the interval [-1,1]. (3 Marks)
- c) Find

i) $\int \frac{v}{1-v} dv$ (3 marks)

ii) $\int \frac{1}{1+\sin x} dx$ (5 marks)

d) Water flows from the bottom of a storage tank at a rate of r(t) = 150 - 5t litres per minute. Find the amount of water that flows from the tank during the first 15 minutes.

(3 marks)

QUESTION TWO (15 MARKS)

a) Evaluate

i)
$$\int_{0}^{\infty} (x+1)e^{x^{2}+2x}dx$$
 (3 marks)

ii) Evaluate
$$\int_{1}^{\pi} \sin x \cos x dx$$
 (3 marks)

b) Find
$$\int \frac{x}{x^2 - 2x - 3} dx$$
 (4 marks)

c) Find the surface area of the portion of the sphere generated by revolving the curve $y = \sqrt{1 - x^2}$, $0 \le x \le \frac{1}{2}$. (5 marks)

SECTION B (39 MARKS): ANSWER ANY THREE QUESTIONS

QUESTION THREE(13 MARKS)

- a) Find the area of the region enclosed by $x = y^2$ and y = x 2 (4 marks)
- b) Derive the volume of a sphere of radius r. (5 marks)
- c) Find $\int \tan^2 x \sec^4 x dx$ (4 marks)

QUESTION FOUR(13 MARKS)

- a) Find $\int \cos^5 x dx$ (3 marks)
- b) Find the volume of the solid generated when the region between the graphs $f(x) = \frac{1}{2} + x^2 \text{ and } g(x) = x \text{ over the interval [1,2] is revolved about the x-axis.}$ (4 marks)
- c) Sketch the curve $y = x^2 + 2$ for $0 \le x \le 3$. Approximate the area under this curve using the trapezium rule with three trapezia. Find exact area and find the percentage error in using trapezium rule. (6 marks)

QUESTION FIVE(13 MARKS)

- a) Use the cylindrical shell to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, x = 1, and x = 4 and the x-axis is revolved about the y-axis. (4 marks)
- b) Find the length of the curve $y = x^{\frac{3}{2}}$ form the point (1,1) to (2,2 $\sqrt{2}$). (5 marks)
- c) Obtain the integral, $\int \ln x dx$ (4 marks)

QUESTION SIX(13 MARKS)

a) Use Simpson's rule to evaluate the integral $\int_{0}^{\infty} \frac{1}{1+x^2} dx$ using 10 strips of equal width. (5 marks)

Main Exam

- b) Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval (1,4) is revolved about the x-axis. (4 marks)
- c) Find $\int \frac{1}{\sqrt{a^2 + x^2}} dx$ for all $a \neq 0$ (4 marks)

QUESTION SEVEN(13 MARKS)

a) Obtain the following integrals

i)
$$\int \frac{x^3 + x}{(x^4 + 2x^2 + 3)^{11}} dx$$
 (3 marks)

ii)
$$\int \frac{\ln(x+1)}{(x+1)} dx$$
 (3 marks)

b) Evaluate the following integrals

i)
$$\int_0^6 \frac{x}{\sqrt{x+1}} dx$$
 (4 marks)

ii)
$$\int x^2 \cos x^3 dx$$
 (3 marks)